EM Waves and Transmission Lines (2-2 ECE, R20, JNTUA)



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Electromagnetic Waves and Transmission Lines (2-2 ECE, R20) TWO MARK QUESTIONS AND ANSWERS UNIT-I (Electrostatics)

1. Define a scalar and vector?

Answer:

A physical quantity having only magnitude is called as scalar. Examples of scalars are Time, Temperature, Mass, Distance, Work, Electric potential, etc. A physical quantity having both magnitude and direction is called as vector. Examples of vectors are Velocity, Displacement, Force, Electric field intensity, Magnetic field intensity, etc.

2. Define Dot product?

Answer:

The dot product between the two vectors is defined as the product of the magnitude of two vectors and cosine of the angle between the two vectors. Let A and B are the two vectors, then the dot product is written as

$$A \cdot B = |A||B|\cos\theta_{AB}$$

The dot product of two vectors is a scalar.

3. Define the Cross product?

Answer:

The cross product between the two vectors is defined as the product of the magnitude of two vectors and sine of the angle between the vectors. Let A and B are the two vectors, then the cross product is written as

$$A \times B = |A||B| \sin \theta_{AB} a_n$$

Where a_n is the unit vector normal to the plane containing the two vectors A and B. The cross reproduct of two vectors is a vector.

4. Write the properties of dot product?

Answer:

i) It obeys the commutative property. That is $A \cdot B = B \cdot A$ ii) It obeys the distributive property. That is $A \cdot (B + C) = A \cdot B + A \cdot C$ $A \cdot A = |A|^2 = A^2$

5. Write the properties of cross product?

Answer:

It does not obey the commutative property. That is

 $A \times B \neq B \times A$

But

$$A \times B = -B \times A$$

It obeys the distributive property. That is

$$A \times (B + C) = A \times B + A \times C$$

It do not obeys the associative property

$$A \times (B \times C) \neq (A \times B) \times C$$

6. Define a position vector?

Answer:

The position vector is defined as the directed distance from the origin to the location of a point. The position vector r_P of point P(1,2,3) can be written as



7. Define a distance vector?

Answer:

The distance vector is defined as the directed distance from one point to another point. The distance vector R between the two points Q(4,5,6) and P(1,2,3) can be written as



8. Transform the point P(1,2,3) in to cylindrical coordinates? Answer:

Given point P(1,2,3) is in Cartesian coordinate systems. i.e. x = 1, y = 2, z = 3.

$$\rho = \sqrt{x^2 + y^2} = \sqrt{1^2 + 2^2} = \sqrt{5} = 2.236$$

$$\emptyset = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{2}{1} = 63.43$$

$$z = z = 3$$

$$P(1,2,3) = P(2.236, 63.43^0, 3)$$

Therefore

9. Transform the point P(1,2,3) in to spherical coordinates?

Answer:

Therefore

Given point P(1,2,3) is in Cartesian coordinate systems. i.e. x = 1, y = 2, z = 3.

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{1^2 + 2^2 + 3^2} = 3.74$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \cos^{-1} \frac{3}{\sqrt{1^2 + 2^2 + 3^2}} = 36.66$$

$$\varphi = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{2}{1} = 63.43$$

$$P(1,2,3) = P(3.74, 36.66^0, 63.43^0)$$

10. Transform the point P(3,60⁰,5) in to Cartesian coordinates? Answer:

Given point P(3,60⁰,5) is in cylindrical coordinate systems. i.e. $\rho = 3$, $\emptyset = 60^{\circ}$, z = 5.

$$x = \rho \sin \varphi = 3 \sin 60 = 2.6$$

$$y = \rho \cos \varphi = 3 \cos 60 = 1.5$$

$$z = z = 5$$

 $P(3,60^{\circ},5) = P(2.6,1.5,5)$ Therefore

11. Transform the point P(3,60⁰,90⁰) in to Cartesian coordinates?

Answer:

Given point P(3,60⁰,90⁰) is in spherical coordinate systems. i.e. r = 3, $\theta = 60^{\circ}$, $\emptyset = 90^{\circ}$.

$$x = r\sin\theta\cos\varphi = 3\sin60\cos90 = 0$$

$$y = r\sin\theta\sin\varphi = 3\sin60\sin90 = 2.6$$

$$z = r\cos\theta = 3\cos 60 = 1.5$$

Therefore
$$P(3,60^{\circ},90^{\circ}) = P(0,2.6,1.5)$$

12. What is the statement of Divergence theorem?

Answer:

The divergence theorem states that, surface integration of any vector field A is equal to the volume integration of divergence of that vector filed. That is,

$$\int A \cdot ds = \int \nabla \cdot A \, dv$$

By using the divergence theorem we can convert the surface integration into volume integration or vice versa.

13. What is the statement of Stokes theorem? Answer:

The stoke's theorem states that, line integration of any vector field A is equal to the surface integration of curl of that vector filed. That is,

$$\int A \cdot dl = \int \nabla \times A \, ds$$

By using the stoke's theorem we can convert the line integration into surface integration or vice versa.

14. If a vector $A = a_x + 2a_y + 3a_z$ find its magnitude and direction? Answer:

$$A = a_x + 2a_y + 3a_z$$

Magnitude

 $|A| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} = 3.742$ $a_A = \frac{A}{|A|} = \frac{a_x + 2a_y + 3a_z}{3.742} = 0.267a_x + 0.534a_y + 0.802a_z$ Direction

15. If $A = 2a_x + 5a_y + 6a_z$ and $B = a_x - 3a_y + 6a_z$ find A+B and A-B?

Answer:

$$A = 2a_x + 5a_y + 6a_z$$

$$B = a_x - 3a_y + 6a_z$$

$$A + B = (2 + 1)a_x + (5 - 3)a_y + (6 + 6)a_z = 3a_x + 2a_y + 12a_z$$

$$A - B = (2 - 1)a_x + (5 + 3)a_y + (6 - 6)a_z = a_x + 8a_y$$

16. If $A = a_x + a_y + 2a_z$ and $B = 2a_x + a_y + a_z$ find A·B? Answer:

 $A \cdot B = (a_x + a_y + 2a_z) \cdot (2a_x + a_y + a_z) = (1)(2) + (1)(1) + (2)(1) = 5$ 17. If $A = 2a_x + a_y + 2a_z$ and $B = a_x + 2a_y + a_z$ find AXB? Answer:

$$A \times B = \begin{vmatrix} a_{x} & a_{y} & a_{z} \\ A_{x} & A_{y} & A_{z} \\ B_{x} & B_{y} & B_{z} \end{vmatrix}$$
$$A \times B = \begin{vmatrix} a_{x} & a_{y} & a_{z} \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$
$$A \times B = a_{x}(1-4) - a_{y}(2-2) + a_{z}(4-1) = -3a_{x} + 3a_{z}$$

18. If $A = 3a_x + 5a_y + 2a_z$ and $B = 6a_x + 2a_y + a_z$ find the angle between two vectors A and B?

Answer:

$$A = 3a_x + 5a_y + 2a_z$$
$$B = 6a_x + 2a_y + a_z$$

From the definition of dot product

$$A \cdot B = |A||B|\cos\theta_{AB}$$

$$A \cdot B = (3a_x + 5a_y + 2a_z) \cdot (6a_x + 2a_y + a_z) = (3)(6) + (5)(2) + (2)(1) = 30$$

$$|A| = \sqrt{3^2 + 5^2 + 2^2} = \sqrt{38} = 6.164$$

$$|B| = \sqrt{6^2 + 2^2 + 1^2} = \sqrt{41} = 6.4$$

$$\theta_{AB} = \cos^{-1}\frac{A \cdot B}{|A||B|} = \cos^{-1}\frac{30}{(6.164)(6.4)} = 40.5 \ degrees$$

19. What is the statement of Coulomb's Law?

Answer:

The statement of Coulomb's Law is, when the two stationary charges Q_1 and Q_2 are separated with a distance R, then the force is existing between the two charges and this force is

- i) In the direction of line joining the two charges.
- ii) Directly proportional to the product of the two charges.
- iii) Inversely proportional to square of the distance between the charges.

There fore

$$F = \frac{Q_1 Q_2}{4\pi \varepsilon R^2} a_R$$

20. Define the electric field intensity? Answer:

Electric field intensity is defined as electric force per unit charge. The electric field intensity is represented with E. E = F/Q.

Therefore

The units of electric field intensity are Newton/coulomb or Volts/meter. Electric field intensity due to point charge Q at a distance of R is given by

$$E = \frac{Q}{4\pi\varepsilon R^2}a_R$$

21. Define electric flux density?

Answer:

Electric flux density is defined as the electric flux per unit area. It is represented with D. Its units are Weber/ m^2 or C/ m^2 . The relation between the electric flux density and electric field intensity is given by

 $D = \varepsilon E$

22. What is the statement of Gauss's Law?

Answer:

The Gauss's Law states that an electric flux passing through any closed surface is equal to the total charge enclosed by that closed surface. i.e.

$$\psi = Q_{enc}$$

Where Q_{enc} is the total charge enclosed by the closed surface.

The Gauss's Law will be used to find out electric flux density due to the symmetric charge distributions.

23. Define electric potential?

Answer:

An electric potential is defined as work per unit charge. It is represented with V. Its units are joules/coulomb or simply volts. The electric potential due to any point charge Q at a distance of R is given by

$$V = \frac{Q}{4\pi\varepsilon R}$$

The electric potential is a scalar quantity.

24. What is the relation between E and V?

Answer:

The relation between E and V is given by

$$E = -\nabla V$$

The above relation says that the potential gradient is nothing but electric field intensity.

The above relation can be used to find out the electric field intensity from the electric potential.

25. What are the two Maxwell's equations for electrostatic fields?

Answer:

The Maxwell's equations for electrostatic fields are given by

Integral form:

$$\int D \cdot ds = \int \rho_v dv$$
$$\int E \cdot dl = 0$$

Point form:

$$\nabla \cdot D = \rho_v$$
$$\nabla \times E = 0$$

26. What is an electric dipole?

Answer:

When the two stationary charges with equal amplitude and opposite phase are separated with a distance 'd' then it is known as electric dipole. The product of the charge and the distance of separation is known as the dipole moment.



Figure 1.30 An electric dipole

27. Define convection current?

Answer:

The flow of charge carriers through the insulating medium is known as convection current. Example of convection current is flowing of electrons through the vacuum of cathode ray tube. The convection current density is given by

$$J_c = \rho_v v_y$$

Where ρ_v is the volume charge density and v_y is the velocity.

28. Define conduction current?

Answer:

The flow of charge carriers through the conducting medium is known as conduction current. The conduction current 'I' can be expressed in terms of current density as

$$I=\int J\cdot ds$$

29. Define dielectric constant?

Answer:

Dielectric constant or relative permittivity (ε_r) of any dielectric medium is defined as the ratio between the permittivity (ε) of the medium and permittivity of free space (ε_0). That is

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

30. Define Isotropic dielectric?

Answer:

A dielectric is said to be isotropic if the dielectric constant (ϵ_r) and conductivity (σ) is constant at each and every direction.

31. Define Homogeneous dielectric? Answer:

A dielectric is said to be homogeneous if the dielectric constant (ϵ_r) and conductivity (σ) is constant at each and every point. Otherwise it is known as inhomogeneous. Atmosphere is an inhomogeneous dielectric.

32. What is the statement of continuity equation?

Answer:

The continuity equation states that the time rate of decrease of charge with in a volume is equal to the net current passing through that closed surface bounded by that volume. That is

$$I = -\frac{dQ}{dt}$$

The continuity equation is given by

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

33. Define relaxation time?

Answer:

Relaxation time is defined as the time required for the charge to reduce its value by the factor of e^{-1} or 37% of its initial value. Relaxation time is given by

$$T_r = \frac{\varepsilon}{\sigma}$$

34. Write poisson's and Laplaces equations in different coordinate systems? Answer:

Poisson's equation is given by

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

In Cartesian coordinate system:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}$$

In cylindrical coordinate system:

$$\frac{1}{\rho} \left[\frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{\partial}{\partial \varphi} \left(\frac{1}{\rho} \frac{\partial V}{\partial \varphi} \right) + \frac{\partial}{\partial z} \left(\rho \frac{\partial V}{\partial \varphi} \right) \right] = -\frac{\rho_v}{\varepsilon}$$

In spherical coordinate system:

$$\frac{1}{r^2 \sin \theta} \left[\frac{\partial}{\partial r} \left(r^2 \sin \theta \frac{\partial V}{\partial r} \right) + \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{\partial}{\partial \varphi} \left(\frac{1}{\sin \theta} \frac{\partial V}{\partial \varphi} \right) \right] = -\frac{\rho_v}{\varepsilon}$$

The Laplace's equation is given by

$$\nabla^2 V = 0$$

In Cartesian coordinate system:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

In cylindrical coordinate system:

$$\frac{1}{\rho} \left[\frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{\partial}{\partial \varphi} \left(\frac{1}{\rho} \frac{\partial V}{\partial \varphi} \right) + \frac{\partial}{\partial z} \left(\rho \frac{\partial V}{\partial \varphi} \right) \right] = 0$$

In spherical coordinate system:

$$\frac{1}{r^2 \sin \theta} \left[\frac{\partial}{\partial r} \left(r^2 \sin \theta \frac{\partial V}{\partial r} \right) + \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{\partial}{\partial \varphi} \left(\frac{1}{\sin \theta} \frac{\partial V}{\partial \varphi} \right) \right] = 0$$

UNIT-II (Magnetostatics)

1. Write the statement of Biot-Savart Law?

Answer:

The Biot-Savart Law states that the magnetic field intensity 'dH' produced by any current element 'Idl' is directly proportional to the product of the current element 'Idl" and sine of the angle α between the current element & the line joining the current element to the point of interest. Also it is inversely proportional to the square of the distance between the current element and point of interest.

$$dH = k \frac{Idl \sin \theta}{R^2} = \frac{Idl \sin \theta}{4\pi R^2} = \frac{Idl \times a_R}{4\pi R^2}$$

2. Write the statement of Ampere's Circuital Law?

Answer:

Ampere's circuital law states that, the line integration of tangential components of magnetic field intensity around any closed path is equal to the current enclosed by that closed path. That is

$$\int H \cdot dl = I_{enc}$$

3. Define magnetic flux density?

Answer:

The magnetic flux density is defined as the magnetic flux per unit area. It is represented with letter B. The units of magnetic flux density are Weber/ m^2 or A/ m^2 . The relation between B and H is given by

$$B = \mu H$$

The magnetic flux (ψ) in terms of B can be written as

$$\psi = \int B \cdot ds$$

4. What are the two Maxwell's equations for magnetostatic fields?

Answer:

The Maxwell's equations for magnetostatic fields are given by

Integral form:

$$\int B \cdot ds = 0$$
$$\int H \cdot dl = \int J \cdot ds$$

Point form:

$$\nabla \cdot B = 0$$
$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

5. What is the magnetic force on the charged particle?

Answer:

When any charged particle of charge 'Q' having a mass 'm' moving with a velocity 'u' in the presence of magnetic field 'B' then the force on the charged particle by the magnetic field is given by

$$F_m = Qu \times B$$

The directions of F, u and B are always perpendicular to each other.

6. What is the Lorentz force equation?

Answer:

When any charged particle of charge 'Q' having a mass 'm' moving with a velocity 'u' in the presence of electromagnetic field, then the total force on the charged particle exerted by both electric field and magnetic field is known as Lorentz force equation and is given by

$$F = QE + Qu \times B = Q(E + u \times B)$$

7. What is the magnetic force on current element?

Answer:

When any current element 'Idl' is placed in the presence of magnetic field 'B' then the force exerted by the magnetic field on the current element is given by

$$F_m = \int I dl \times B$$

8. What is the ampere's force Law?

Answer:

The magnetic force existing between the two current elements or current loops can be expressed as

The force on current element 1 by current element 2 is given by

$$F_1 = \frac{\mu}{4\pi} \int \int I_1 dl_1 \times I_2 dl_2 \times a_{R21}$$

Similarly the force on current element 2 by current element 1 is given by

$$F_2 = \frac{\mu}{4\pi} \int \int I_2 dl_2 \times I_1 dl_1 \times a_{R12}$$

The above two equations represents the ampere's force law.

9. Define magnetic torque?

Answer:

The mechanical moment of force is nothing but the magnetic torque. It is represented with 'T'. The magnetic torque is also defined as the cross product between the moment arm(r) and the magnetic force (F).

Therefore

$$T = r \times F$$

The relation between the magnetic torque (T) and magnetic moment (m) is given by

 $T = m \times F$

10. Define magnetic dipole moment?

Answer:

The product of the current and the area of the current loop is known as the magnetic dipole moment. Therefore the magnetic dipole moment is given by

$$m = ISa_n$$

Where 'I' is the current in the circuit 'S' is the area of the circuit and a_n is the unit vector normal to the current loop. The direction of 'm' is always perpendicular or normal to the current loop. The relation between the magnetic torque (T) and magnetic moment (m) is given by

$$T = m \times F$$

11. What is magnetic dipole?

Answer:

A magnet bar or any filamentary current loop is usually referred to as a magnetic dipole. The magnetic dipole is shown in the following figure.



Fig:Magnetic dipole

The magnetic flux density and magnetic field intensity due to the magnetic dipole is given by

$$B = \frac{\mu m}{4\pi r^3} (2\cos\theta \, a_r + \sin\theta \, a_\theta)$$
$$H = \frac{m}{4\pi r^3} (2\cos\theta \, a_r + \sin\theta \, a_\theta)$$

12. Define magnetic scalar potential?

Answer:

The magnetic scalar potential is defined as the scalar potential produced by the magnetic field. From the electrostatics we know that the relation between E and V as $E = -\nabla V$ Similarly in magnetostatics we can have the relation

$$H = -\nabla V_m \qquad if J = 0$$

In above equation the letter V_m represents the magnetic scalar potential. Its units are amperes. In above equation the condition J=0 represents that, the magnetic scalar potential should be evaluated wherever the current density (J) is zero.

13. Define magnetic vector potential?

Answer:

We know that, from maxwell's equations

 $\nabla \cdot B = 0$

Also from the standard vector identities, the divergence of curl of any vector quantity is equal to zero. i.e.

$$\nabla \cdot (\nabla \times A) = 0$$

By comparing the above two equations we can say that,

$$B = \nabla \times A$$

In above equation vector 'A' is called as the magnetic vector potential. The magnetic vector potential due to different current distributions can be written as

$$A = \int \frac{\mu I dl}{4\pi R} \qquad due \ to \ line \ current$$
$$A = \int \frac{\mu k ds}{4\pi R} \qquad due \ to \ sheet \ current$$
$$A = \int \frac{\mu J dv}{4\pi R} \qquad due \ to \ volume \ current$$

14. Define self inductance? Answer: When a current I is applied to any closed circuit contains 'N' no.of turns, then this current produces a magnetic flux (Ψ) which intern flows through the each and every turn. This flux linkage (λ) is given by

Also the flux linkage proportional to the current I, i.e. $\lambda \alpha I$ Therefore

 $\lambda = LI$

 $\lambda = N\Psi$

In above relation the letter L represents proportional constant and is called as the self inductance.

$$L = \frac{\lambda}{I} = \frac{N\Psi}{I}$$

15. Define mutual inductance?

Answer:

The inductance between the two closed circuits will be represented with mutual inductance. When a current I_1 and I_2 is applied to closed circuits 1 and 2 contains 'N₁' and 'N₂' no.of turns, then the current I_2 produces a magnetic flux (Ψ_{12}) at the circuit 1 which intern flows through the each and every turn of circuit 1. This flux linkage (λ_{12}) is given by

$$\mathfrak{l}_{12} = N_1 \Psi_{12}$$

Also the flux linkage proportional to the current I_2 , i.e. $\lambda_{12} \; \alpha \; I_2$ Therefore

$$\lambda_{12} = M_{12}I_2$$

In above relation the letter M_{12} represents proportional constant and is called as the mutual inductance.

$$M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \Psi_{12}}{I_2}$$

16. Define magnetic energy?

Answer:

Energy stored in a magnetostatic field is known as the magnetic energy. The magnetic energy can be obtained from the fundamentals of energy stored in an inductor. Therefore the magnetic energy (w_m) stored in a magnetostatic field is given by

$$w_m = \frac{1}{2} \int B \cdot H \, dv = \frac{1}{2} \int \mu H^2 dv = \frac{1}{2} \int \frac{B^2}{\mu} dv$$

The magnetic energy density is defined as the magnetic energy per unit volume and is given by

$$w_{md} = \frac{1}{2}(B \cdot H) = \frac{1}{2}(\mu H^2) = \frac{1}{2}\left(\frac{B^2}{\mu}\right)$$

17. Write the statement of Faraday's law?

Answer:

Faraday's law states that, the time varying magnetic flux could able to produce the e.m.f. or voltage with in a closed circuit. i.e.

$$V = -N\frac{d\Psi}{dt}$$

Where N is the number of turns in the circuit, Ψ is the magnetic flux. The integral form of Faraday's law is given by

$$\int E \cdot dl = -\int \frac{\partial B}{\partial t} \cdot ds$$

The point form of Faraday's law is given by

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

18. What is meant by transformer e.m.f? Answer:

An electro motive force (e.m.f) induced in a stationary loop by the time varying magnetic field is called as the transformer e.m.f. The transformer e.m.f is given by

$$V_{e.m.f} = -N \frac{d\Psi}{dt}$$

19. What is the insistency of Ampere's circuital law?

Answer:

From the Ampere's circuital law we know that,

$$\nabla \times H = J$$

If we take del with dot on both sides of above equation, then it becomes

$$\nabla \cdot \nabla \times H = \nabla \cdot J$$

$$0 = \nabla \cdot J$$

Because from the vector results it is known that, the divergence of curl of any vector quantity is equal to zero.

Form the above relation we can say that divergence of J is zero, but from the continuity equation the divergence of J must be some finite value. It should not be zero. Therefore in the statement of ampere's circuital law there must be some other parameter in addition to the conduction current density (J).

20. Define displacement current density? Answer:

The inconsistency of ampere's circuital law says that, in addition to the conduction current density (J) there is one more current density called displacement current density (J_d) which can be defined as the rate of change of electric displacement (D).

From the modified Ampere's circuital law we know that,

$$\nabla \times H = J + J_d = J + \frac{\partial D}{\partial t}$$

In above equation J_d is known as the displacement current density. Therefore

$$J_d = \frac{\partial D}{\partial t}$$

21. Write the Maxwell's equations for time varying fields in integral form? Answer:

Integral from of Maxwell's equations can be written as

$$\int \mathbf{B} \cdot d\mathbf{s} = 0$$
$$\int D \cdot ds = \int \rho_v dv$$
$$\int E \cdot dl = -\int \frac{\partial B}{\partial t} \cdot ds$$
$$\int H \cdot dl = \int \left(J + \frac{\partial D}{\partial t}\right) \cdot ds$$

22. Write the Maxwell's equations for time varying fields in point form or differential form?

Answer:

Point form or differential from of Maxwell's equations can be written as

$$\nabla \cdot B = 0$$
$$\nabla \cdot D = \rho_{v}$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

23. Write the Maxwell's equations for time varying fields in Phasor form? Answer:

Phasor form of Maxwell's equations can be written as

$$\nabla \cdot B = 0$$

$$\nabla \cdot D = \rho_{v}$$

$$\nabla \times E = -j\omega B = -j\omega\mu H$$

$$\nabla \times H = J + j\omega D = \sigma E + j\omega \varepsilon E = (\sigma + j\omega)E$$

24. Give the word statement of Maxwell's equations for time varying fields? Answer:

The four Maxwell's equations for time varying fields can be written as

$$\int \mathbf{B} \cdot d\mathbf{s} = 0$$
$$\int D \cdot ds = \int \rho_{v} dv$$
$$\int E \cdot dl = -\int \frac{\partial B}{\partial t} \cdot ds$$
$$\int H \cdot dl = \int \left(J + \frac{\partial D}{\partial t}\right) \cdot ds$$

The first Maxwell's equation represents that, the magnetic fields are always in the form of closed loops or the magnetic poles or charges cannot be separated.

The second Maxwell's equation represents that, the electric flux passing through any closed surface is equal to the total charge enclosed by that closed surface.

The third Maxwell's equation represents that; time varying magnetic field could able to produce the e.m.f in a closed circuit or the line integration of electric field intensity (E) is equal to the surface integration of time rate of decrease of magnetic flux density (B).

The fourth Maxwell's equation represents that, the line integration of magnetic field intensity is equal to the surface integration of conduction current density (J) plus displacement current density (J_d) .

<u>UNIT-III</u>

1. List out the electric boundary conditions at the interface of dielectric-dielectric? Answer:

The four electric boundary conditions at the interface of dielectric-dielectric are given by

1. The tangential components of electric field intensity are continuous across the dielectric-dielectric boundary interface. i.e.

$$E_{1t} = E_{2t}$$

2. The tangential components of electric flux density are discontinuous across the dielectric-dielectric boundary interface. i.e.

 $D_{1t} \neq D_{2t}$

3. The normal components of electric flux density are continuous across the dielectricdielectric boundary interface. i.e.

$$D_{1n} = D_{2n}$$

4. The normal components of electric field intensity are discontinuous across the dielectric-dielectric boundary interface. i.e.

$$E_{1n} \neq E_{2n}$$

2. List out the electric boundary conditions at the interface of dielectric-conductor? Answer:

The four electric boundary conditions at the interface of dielectric-conductor are given by

1. The tangential components of electric field intensity are continuous across the dielectric- conductor boundary interface. i.e.

$$E_{1t} = E_{2t}$$

2. The tangential components of electric flux density are continuous across the dielectric- conductor boundary interface. i.e.

$$D_{1t} = D_{2t}$$

3. The normal components of electric flux density are continuous across the dielectricconductor boundary interface. i.e.

$$D_{1n} \neq D_{2n}$$

4. The normal components of electric field intensity are discontinuous across the dielectric- conductor boundary interface. i.e.

$$E_{1n} \neq E_{2n}$$

3. List out the magnetic boundary conditions at the interface of dielectric-dielectric? Answer:

The four magnetic boundary conditions at the interface of dielectric-dielectric are given by

1. The tangential components of magnetic field intensity are continuous across the dielectric-dielectric boundary interface. i.e.

$$H_{1t} = H_{2t}$$

2. The tangential components of magnetic flux density are discontinuous across the dielectric- dielectric boundary interface. i.e.

$$B_{1t} \neq B_{2t}$$

3. The normal components of magnetic flux density are continuous across the dielectricdielectric boundary interface. i.e.

$$B_{1n} = B_{2n}$$

4. The normal components of magnetic field intensity are discontinuous across the dielectric-dielectric boundary interface. i.e.

$$H_{1n} \neq H_{2n}$$

4. List out the magnetic boundary conditions at the interface of dielectric-conductor? Answer:

The four magnetic boundary conditions at the interface of dielectric-conductor are given by

1. The tangential components of magnetic field intensity are discontinuous across the dielectric-conductor boundary interface. i.e.

$$H_{1t} \neq H_{2t}$$

2. The tangential components of magnetic flux density are discontinuous across the dielectric-conductor boundary interface. i.e.

$$B_{1t} \neq B_{2t}$$

3. The normal components of magnetic flux density are continuous across the dielectricconductor boundary interface. i.e.

$$B_{1n} = B_{2n}$$

4. The normal components of magnetic field intensity are continuous across the dielectric-conductor boundary interface. i.e.

$$H_{1n} = H_{2n}$$

5. Write the wave equation for conducting media?

Answer:

The wave equation or Helmholtz equation for conducting media is given by

$$\nabla^{2}E = \mu\sigma\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^{2}E}{\partial t^{2}}$$
(OR)
$$\nabla^{2}H = \mu\sigma\frac{\partial H}{\partial t} + \mu\varepsilon\frac{\partial^{2}H}{\partial t^{2}}$$

The wave equation will be used to find out the characteristics of a wave when it is propagating through the particular media.

6. Write the wave equation for perfect dielectric media?

Answer:

The wave equation or Helmholtz equation for perfect dielectric media is given by

$$\nabla^{2}E = \mu \varepsilon \frac{\partial^{2}E}{\partial t^{2}}$$
(OR)
$$\nabla^{2}H = \mu \varepsilon \frac{\partial^{2}H}{\partial t^{2}}$$

The wave equation will be used to find out the characteristics of a wave when it is propagating through the particular media.

7. Define uniform plane wave?

Answer:

A plane wave is defined as a wave which maintains constant phase at each and every point in the space. A uniform plane wave is defined as the wave which maintains constant value of amplitude and phase at each and every point in the space.

8. List out the relations between E and H?

Answer:

All relations between electric field intensity (E) and magnetic field intensity (H) are given by

- (i) Always E and H are pendicular.
- (ii) The ratio between E and H gives the intrinsic impedance of media. i.e.

$$\frac{E}{H} = \eta$$

(iii) The cross product between E and H gives the power flow per unit area. i.e

$$E \times H =$$

9. Write the differences between the conductors and dielectrics?

Answer:

A perfect or good conductor maintains infinite value of conductivity (σ) and zero resistivity (ρ), where as a perfect dielectric maintains infinite value of resistivity (ρ) and zero value of conductivity (σ). A conductor can have free charge carriers where as the dielectric do not have free charge carriers. When the ratio between the conduction current density (J_c) and displacement

current density (J_d) is high then it is known as conductor. When this ratio is less than one then it is known as dielectric.

10. Define skin depth or depth of penetration?

Answer:

The depth or distance in a conductor at which the magnitude of waves reduces by the factor of e^{-1} or 37% of its initial value. It is represented with δ . The units of skin depth are meters or centimeters. The skin depth can be expressed as

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

The skin depth is a measure of depth or distance to which an EM wave can penetrate the conducting media.

11. Define polarization?

Answer:

Polarization is defined as the time varying behavior of electric field strength vector at a particular point in the free space. The polarization represents the direction or orientation of electric field strength vector (E). There are different types of polarizations such as linear polarization, elliptical polarization, and circular polarization.

12. Define linear polarization?

Answer:

A polarization is said to be linear if the direction of resultant electric vector is linearly vary with respect to time. Examples of linear polarization are vertical polarization and horizontal polarization. In vertical polarization the direction of E is vertical and where as in horizontal polarization the direction of E is horizontal.

13. Define Elliptical polarization?

Answer:

When the two components of electric field having unequal amplitudes and any phase difference (other than zero), then the resultant vector will trace an elliptical path and hence the resultant polarization is said to be elliptical polarization.

14. Define circular polarization?

Answer:

When the two components of electric field having equal amplitudes and 90⁰ phase difference, then the resultant vector will trace a circular path and hence the resultant polarization is said to be circular polarization. In circular polarization there are two types such as a left circular polarization and right circular polarization.

15. Define vertical or parallel polarization?

Answer:

If the direction of electric vector (E) is perpendicular to the boundary surface or parallel to the plane of incidence, then it is known as vertical or parallel polarization. It is also defined as the polarization in which the direction of magnetic vector (H) is parallel to the boundary interface or perpendicular to the plane of incidence.

16. Define horizontal or perpendicular polarization?

Answer:

If the direction of electric vector (E) is parallel to the boundary surface or perpendicular to the plane of incidence, then it is known as horizontal or perpendicular polarization. It is also defined as the polarization in which the direction of magnetic vector (H) is perpendicular to the boundary interface or parallel to the plane of incidence.

UNIT-IV

1. Define standing waves?

Answer:

The combination of incident and reflected waves gives raise new phenomena called standing waves. The standing waves do not progress but their amplitude will vary with distance and time. When the EM waves are normally incident on a perfect conductor, then the equation for E and H are given by

$$E_T(x, t) = 2E_i \sin \beta x \sin \omega t$$

$$H_T(x, t) = 2H_i \cos \beta x \cos \omega t$$

The above two equations represents the standing waves.

2. Define normal and oblique incidence?

Answer:

When a signal is incidence perpendicularly or normally on a boundary interface then it is known as normal incidence. Similarly when the signal is incidence obliquely or incidence with some angle of incidence other than normal direction, then it is known as oblique incidence.

3. Define direction cosines?

Answer:

When the signal is incident or reflects at some arbitrary direction other than the coordinate directions, then the direction cosines will be used to express the waves. The direction cosines will be in terms of cos and hence these are called as the direction cosines.



The reflected signal shown in the above figure can be expressed with direction cosines as:

 $r \cdot \hat{n} = x \cos A + y \cos B + z \cos C$

4. Define Brewster angle?

Answer:

The Brewster angle is defined as the angle of incidence for which no reflection occurs. In case of parallel or vertical polarization the Brewster angle is given by

$$\theta_1 = \tan^{-1} \left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \right)$$

5. Define total internal reflection?

Answer:

The propagation of waves between the two boundaries with successive internal reflection is known as total internal reflection. The total internal reflection occurs when the angle of incidence is satisfies the critical angle of incidence. The critical angle of incidence is given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Where n_1 and n_2 are the refractive indices of the two media.

6. Define surface impedance?

Answer:

The surface mpedance is defined as the ratio between the tangential components of E and surface current density. i.e.

$$Z_s = \frac{E_{tan}}{J_s}$$

Where E_{tan} is the tangential component of electric field intensity (E) and J_s is the surface current density. The surface impedance will be defined in case of good conductors.

The surface impedance in terms of conductivity (σ) can be expressed as

$$Z_s = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

7. Define surface resistance or skin effect resistance? Answer:

The surface impedance will be defined in case of good conductors. The surface impedance in terms of conductivity (σ) can be expressed as

$$Z_s = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

In general the impedance can be expressed as

$$Z = R_s + jX_s$$

Therefore the skin effect resistance or surface resistance is given by

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

8. Write the statement of poynting theorem?

Answer:

The poynting theorem states that, the vector product of electric field vector E and magnetic field vector H at any point is the measure of rate of energy flow or power flow per square meter at that point. Therefore

$P = E \times H$

Where P is called as the poynting vector. Its units are watts per square meter. The direction of P is always perpendicular the directions of E and H.

UNIT-V (Transmission Lines)

1. List out the types of transmission lines?

Answer:

Transmission lines will be used to transfer the information form one device to other device. The following are the basic types of transmission lines

- (i) Coaxial cable
- (ii) Two-wire line
- (iii) Parallel plate or planar line
- (iv) A wire above the conducting plane
- (v) Microstrip line

2. What are the primary parameters or primary constants of a transmission lines? Answer:

The primary constants of a transmission lines are Resistance(R), Inductance (L), Conductance (G) and Capacitance(C). These parameters are defined as follows:

Resistance: It is defined as a loop resistance per unit length. It is sum of the resistances of both the wires per unit length and its units are ohms / km.

Inductance: It is defined as a loop inductance per unit length. It is sum of the inductances of both the wires per unit length and its units are henries / km.

Conductance: It is defined as the shunt conductance between the two conductors per unit length and its units are mhos / km.

Capacitance: It is defined as the shunt capacitance between the two conductors per unit length and its units are farads / km.

3. Discuss about the secondary parameters of a transmission line ? Answer:

The characteristic impedance (Z_0) and propagation constant (γ) are known as secondary parameters of a transmission line. The equations for Z_0 and γ are given by

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + J\omega C}}$$
$$\gamma = \sqrt{(R + j\omega L)(G + J\omega C)}$$

For lossless line,

$$Z_0 = \sqrt{\frac{L}{C'}} \quad \gamma = j\omega\sqrt{LC}$$

4. Draw the equivalent circuit of lossy transmission line? Answer:

The equivalent circuit of transmission line under lossy mode is shown in figure below.



5. Draw the equivalent circuit of lossless transmission line?

Answer:

The equivalent circuit of lossless transmission line is shown in figure below.



6. Discuss the input impedance of a transmission line? Answer:

The input impedance of a lossy transmission line is given by

$$Z_S = \frac{V_S}{I_S} = \frac{Z_0 (Z_R \cos h\gamma l + Z_0 \sin h\gamma l)}{Z_0 \cos h\gamma l + Z_R \sin h\gamma l}$$

Divide numerator and denominator with $\cos h\gamma l$

$$Z_{S} = \frac{Z_{0}(Z_{R} + Z_{0} \tan h\gamma l)}{Z_{0} + Z_{R} \tan h\gamma l}$$

The input impedance of lossless transmission line is given by

$$Z_{S} = \frac{Z_{0}(Z_{R} + Z_{0} \tan hj\beta l)}{Z_{0} + Z_{R} \tan hj\beta l}$$
$$Z_{S} = \frac{Z_{0}(Z_{R} + jZ_{0} \tan \beta l)}{Z_{0} + jZ_{R} \tan \beta l}$$

7. What do you mean by lossless transmission line and give the condition for lossless line?

Answer:

A transmission line without any losses is known as lossless transmission line. The primary parameters resistance (R) and Conductance (G) are known as lossy elements. The condition for lossless line is given by

$$R = 0, G = 0$$

For lossless transmission line,

Attenuation constant (α) = 0

Phase constant (
$$\beta$$
) = $\omega \sqrt{LC}$

8. Discuss about distortionless transmission line and give the condition for distortionless line

Answer:

A transmission line is said to be distortionless line, if its attenuation constant (α) is independent of the frequency. The condition for the distortionless line is given by

$$\frac{R}{L} = \frac{G}{C}$$

The attenuation constant of distortionless line is given by

$$\alpha = \sqrt{RG}$$

The phase constant of distortionless line is given by

$$\beta = \omega \sqrt{LC}$$

The characteristic impedance of distortionless line is given by

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$$

9. What are the impedances at voltage maximum and voltage minimum of transmission line

Answer:

When the impedance is measured at the point of voltage maximum of standing waves, then it is known as impedance maximum. The equation for the impedance maximum is given by

$$Z_{max} = Z_0 \times SWR$$

The equation for the impedance minimum is given by

$$Z_{min} = Z_0 / SWR$$

Where SWR is Standing Wave Ratio.

10. Define standing wave ratio?

Answer:

The standing wave ratio (SWR) is defined as the ratio between maximum voltage (V_{max}) and minimum voltage (V_{min}). That is

$$SWR = \frac{V_{max}}{V_{min}}$$

When the SWR is expressed in terms of voltage, then it is known as Voltage Standing Wave Ratio (VSWR). Similarly when the SWR is expressed in terms of current, then it is known as Current Standing Wave Ratio (CSWR).

11. Define the reflection coefficient and give the relation between reflection coefficient and standing wave ratio ?

Answer:

Reflection coefficient (Γ) is defined as the ratio between reflected signal to the incident signal. When the reflection coefficient is expressed in terms of voltage, then it is known as voltage reflection coefficient. Similarly if it is expressed in terms of current, then it is known as current reflection coefficient. Therefore,

$$\Gamma = \frac{V_r}{V_i} = -\frac{I_r}{I_i}$$

Where V_r represent the reflected voltage, V_i is the incident voltage, I_r is the reflected current and I_i is the incident current.

The relation between the reflection coefficient (Γ) and standing wave ratio is given by

$$\Gamma = \frac{SWR - 1}{SWR + 1}$$
$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

12. Discuss about the input impedance of microstrip transmissionlines? Answer:

The approximate equation for the characteristic impedance of a microstrip line is given by

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left(\frac{8h}{w} + \frac{w}{4h}\right), & when w/h \le 1\\ \frac{1}{\sqrt{\varepsilon_{eff}}} \frac{120\pi}{\left[\frac{w}{h} + 1.393 + 0.667 \ln\left(\frac{w}{h} + 1.444\right)\right]}, & when w/h \ge 1\end{cases}$$

Where ε_{eff} is effective dielectric constant or effective relative permittivity, 'h' is the height of the substrate and 'w' is the width of the conducting strip.

$$\varepsilon_{eff} = \frac{(\varepsilon_r + 1)}{2} + \frac{(\varepsilon_r - 1)}{2\sqrt{1 + 12h/w}}$$

13. Discuss about the applications of transmission lines? Answer:

The applications of transmission lines are given by

- (i) Quarter wave transformer (Matching).
- (ii) Single stub tuner (matching).
- (iii) Slotted line (Impedance measurement).

(iv) Transmission line as circuit elements.

A transmission line having a length $\lambda/4$ can be used for impedance matching. A small section of short circuited transmission line (Stub) can also be used for impedance matching. A transmission line can be used to construct slotted line which can be used for the measurement of unknown impedance.

14. Discuss about applications of smith chart?

Answer:

Smith chart is the most graphical technique used to find the characteristics of a transmission line. Smith chart is also known as polar impedance diagram. The applications of smith chart are given by

- (i) Admittance calculation
- (ii) Calculation of SWR along with impedance or admittance.
- (iii) Calculation of length of short circuited piece of transmission line to give required capacitive or inductive reactance.
- (iv) Calculation of length and position of the single stub and double stubs.

ESSAY TYPE QUESTIONS AND ANSWERS UNIT-I(Electrostatic Fields)

1. State and explain Coulomb's Law?

Answer: The Coulomb's law states that, the force (F) in between two stationary charges Q_1 and Q_2 is

- (i) Along the line joining the two charges
- (ii) Directly proportional to the product of Q_1 and Q_2
- (iii) Inversely proportional to the square of distance (R) between the two charges.

Therefore coulomb's law can be expressed mathematically as

$$F \propto \frac{Q_1 Q_2}{R^2}$$
$$F = k \frac{Q_1 Q_2}{R^2}$$

Where k is known as proportionality constant which is equal to $1/4\pi\epsilon_0$ to satisfy the SI units.

$$\varepsilon_0 = 8.854 \times 10^{-12} F/m$$
$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} a_R - 1$$

Now let us express equation 1 in terms of position vectors from the following figure.



Figure 1.17 Force between two charges

The force on Q_2 by charge Q_1 is given by

$$F_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2} a_{R_{12}} - 2$$

The unit vector of R_{12} is given by

$$a_{R_{12}} = \frac{R_{12}}{|R_{12}|} - 3$$

Substitute equation 3 in equation 2

$$F_{12} = \frac{Q_1 Q_2 R_{12}}{4\pi\varepsilon_0 |R_{12}|^3} - 4$$

The distance vector R_{12} is given by

$$R_{12} = r_2 - r_1 \qquad -5$$

Substitute equation 5 in equation 4

$$F_{12} = \frac{Q_1 Q_2 (r_2 - r_1)}{4\pi\varepsilon_0 |r_2 - r_1|^3}$$

Similarly the equation for force on charge Q_1 by charge Q_2 is given by

$$F_{21} = \frac{Q_1 Q_2 (r_1 - r_2)}{4\pi\varepsilon_0 |r_1 - r_2|^3}$$

The force due to 'N' no.of point charges $(Q_1, Q_2, Q_3, \dots, Q_N)$ on charge Q is given by

$$F = \frac{QQ_1(r-r_1)}{4\pi\varepsilon_0 |r-r_1|^3} + \frac{QQ_2(r-r_2)}{4\pi\varepsilon_0 |r-r_2|^3} + \frac{QQ_3(r-r_3)}{4\pi\varepsilon_0 |r-r_3|^3} + \dots + \frac{QQ_N(r-r_N)}{4\pi\varepsilon_0 |r-r_N|^3}$$
$$F = \frac{Q}{4\pi\varepsilon_0} \sum_{k=1}^N \frac{Q_k(r-r_k)}{|r-r_k|^3}$$

The electric field intensity (E) is defined as force per unit charge. i.e.

$$E = \frac{F}{Q}$$

The electric field intensity due to 'N' no.of point charges at any interesting point is given by

$$E = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k(r - r_k)}{|r - r_k|^3}$$

2. Find out the electric field intensity due to infinite length line charge ?

Answer: The electric field intensity due to infinite length line charge is obtained from the following figure.



Figure 1.18 EFI due to line charge

The general equation for electric field intensity due line charge is given by

$$E = \int \frac{\rho_l dl}{4\pi\varepsilon_0 |R|^2} a_R \qquad -1$$

From figure,

$$dl = dz - 2$$

$$R = \rho a_{\rho} - z a_{z}$$

$$|R| = \sqrt{\rho^{2} + z^{2}} - 3$$

$$a_{R} = \frac{R}{|R|} = \frac{\rho a_{\rho} - z a_{z}}{\sqrt{\rho^{2} + z^{2}}} - 4$$

Substitute equations 2, 3 and 4 in equation 1

$$E = \int \frac{\rho_l dz}{4\pi\varepsilon_0 \left(\sqrt{\rho^2 + z^2}\right)^2} \cdot \left(\frac{\rho a_\rho - za_z}{\sqrt{\rho^2 + z^2}}\right)$$

Due to symmetry of line charge, the z-components of E will be cancelled. That is za_z will be zero.

$$E = \int \frac{\rho_l a_2 \rho a_{\rho}}{4\pi\varepsilon_0 (\rho^2 + z^2)^{3/2}}$$
$$E = \frac{\rho_l \rho}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{1}{(\rho^2 + z^2)^{3/2}} dz \, a_{\rho} = \frac{\rho_l \rho}{4\pi\varepsilon_0} (2) \int_{0}^{L/2} \frac{1}{(\rho^2 + z^2)^{3/2}} dz \, a_{\rho} \qquad -5$$

But

$$\int \frac{1}{\left(a^2 + x^2\right)^{3/2}} dx = \frac{x}{a^2 \sqrt{a^2 + x^2}}$$

Therefore equation 5 becomes

$$E = \frac{\rho_l \rho}{2\pi\epsilon_0} \left[\frac{z}{\rho^2 \sqrt{\rho^2 + z^2}} \right]_0^{L/2} a_\rho = \frac{\rho_l \rho}{2\pi\epsilon_0 \rho^2} \left[\frac{L/2}{\sqrt{\rho^2 + (L/2)^2}} \right] a_\rho$$

$$E = \frac{\rho_l}{2\pi\varepsilon_0\rho} \left[\frac{L}{\sqrt{4\rho^2 + L^2}} \right] a_\rho$$

When $L \rightarrow \infty$, then $4\rho^2$ can be neglected.

$$E = \frac{\rho_l}{2\pi\varepsilon_0\rho} \left[\frac{L}{\sqrt{L^2}}\right] a_\rho = \frac{\rho_l}{2\pi\varepsilon_0\rho} a_\rho$$

3. A circular ring of radius 'a' carries a uniform charge ρ_L C/m and is placed on the xy-plane with axis same as the z-axis. Show that

$$E(0,0,h) = \frac{\rho_L a n}{2\varepsilon_0 [h^2 + a^2]^{3/2}} a_2$$

Answer:



The general equation for electric field intensity due line charge is given by

$$E = \int \frac{\rho_l dl}{4\pi\varepsilon_0 |R|^2} a_R \qquad -1$$

From figure,

$$dl = \rho d\varphi = ad\varphi - 2$$

$$R = ha_z - aa_\rho$$

$$|R| = \sqrt{h^2 + a^2} - 3$$

$$a_R = \frac{R}{|R|} = \frac{ha_z - aa_\rho}{\sqrt{h^2 + a^2}} - 4$$

Substitute equations 2, 3 and 4 in equation 1

$$E = \int \frac{\rho_L a d\varphi}{4\pi\varepsilon_0 \left(\sqrt{h^2 + a^2}\right)^2} \cdot \left(\frac{ha_z - aa_\rho}{\sqrt{h^2 + a^2}}\right)$$

Due to symmetry of line charge, the ρ -components of E will be cancelled. That is aa_{ρ} will be zero.

$$E = \int \frac{\rho_L a d\varphi h a_z}{4\pi\varepsilon_0 (h^2 + a^2)^{3/2}} = \frac{\rho_L a h}{4\pi\varepsilon_0 (h^2 + a^2)^{3/2}} \int_0^{2\pi} d\varphi \, a_z$$
$$E = \frac{\rho_L a h}{4\pi\varepsilon_0 (h^2 + a^2)^{3/2}} (2\pi) a_z = \frac{\rho_L a h}{2\varepsilon_0 (h^2 + a^2)^{3/2}} a_z$$

4. A circular disk of radius 'a' is uniformly charged with $\rho_s C/m^2$. The disk lies on the z = 0 plane with axis along the z-axis. Show that at point (0,0,h)



Answer:

The general equation for electric field intensity due to sheet charge is given by

 $E = \int \frac{\rho_S ds}{4\pi\varepsilon_0 |R|^2} a_R \qquad -1$

From figure,

The distance vector 'R' is given by

$$R = ha_z - \rho a_\rho$$

$$|R| = \sqrt{h^2 + \rho^2} - 2$$

$$a_R = \frac{R}{|R|} = \frac{ha_z - \rho a_\rho}{\sqrt{h^2 + \rho^2}} - 3$$

4

The differential surface area along z-direction is given by

$$ds = \rho d\rho d\varphi$$
 –

Substitute equations 2, 3 and 4 in equation 1

$$E = \int \frac{\rho_{S}(\rho d\rho d\varphi)}{4\pi\varepsilon_{0} \left(\sqrt{h^{2} + \rho^{2}}\right)^{2}} \frac{ha_{z} - \rho a_{\rho}}{\sqrt{h^{2} + \rho^{2}}}$$
$$E = \int \frac{\rho_{S}(\rho d\rho d\varphi) [ha_{z} - \rho a_{\rho}]}{4\pi\varepsilon_{0} (h^{2} + \rho^{2})^{3/2}}$$

Due to symmetry of circular disk, the ρ -components of E will be cancelled. i.e. $\rho a_{\rho} = 0$

$$E = \int \frac{\rho_{S}(\rho d\rho d\varphi)[ha_{z}]}{4\pi\varepsilon_{0}(h^{2} + \rho^{2})^{3/2}}$$
$$E = \frac{\rho_{S}h}{4\pi\varepsilon_{0}} \int_{0}^{a} \frac{\rho}{(h^{2} + \rho^{2})^{3/2}} d\rho \int_{0}^{2\pi} d\varphi \quad a_{Z}$$
$$E = \frac{\rho_{S}h}{4\pi\varepsilon_{0}} \left[\frac{-1}{\sqrt{h^{2} + \rho^{2}}}\right]_{0}^{a} [\varphi]_{0}^{2\pi} \quad a_{Z} \qquad because \int \frac{x}{\sqrt{(a^{2} + x^{2})^{3/2}}} = \frac{-1}{\sqrt{a^{2} + x^{2}}}$$

$$E = \frac{\rho_S h}{4\pi\varepsilon_0} \left[\frac{-1}{\sqrt{h^2 + a^2}} + \frac{1}{\sqrt{h^a}} \right] [2\pi] a_Z$$
$$E = \frac{\rho_S}{2\varepsilon_0} \left(\frac{h}{h} - \frac{h}{\sqrt{h^2 + a^2}} \right) a_Z$$
$$E = \frac{\rho_S}{2\varepsilon_0} \left\{ 1 - \frac{h}{[h^2 + a^2]^{1/2}} \right\} a_Z$$

5. State and explain Gauss's Law? Answer:



Figure 1.22 Closed surface

Gauss's Law states that, the total electric flux passing through any closed surface is equal to toal charge enclosed by that closed surface.

$$\psi = Q_{enc} - 1$$

Where ψ represents the total electric flux and Q_{enc} represents the total charge enclosed by the closed surface.

We know that,

$$\psi = \int D \cdot ds \qquad -2$$

And

$$Q_{enc} = Q = \int \rho_v dv \qquad -3$$

Substitute equation 2 and 3 in equation 1

$$\int D \cdot ds = \int \rho_v dv \qquad -4$$

The above equation represent the integral form of Gauss's law By using divergence theorem

$$\int D \cdot ds = \int \nabla \cdot D \, dv \qquad -5$$

Substitute equation 5 in equation 4

$$\int \nabla \cdot D \, dv = \int \rho_v dv$$
$$\nabla \cdot D = \rho_v \qquad -6$$

The above equation represents the point form of Gauss's law

- 6. If $D = (2y^2 + z)a_x + 4xy a_y + x a_z C/m^2$, find
 - (a) Volume charge density at (-1,0,3)
 - (b) The flux through the cube defined by $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$
 - (c) The total charge enclosed by the cube

Answer:

Given

$$D = (2y^2 + z)a_x + 4xy a_y + x a_z C/m^2$$

(a)

$$\rho_v = \nabla \cdot D$$

Express in Cartesian coordinate system

(b)

$$\rho_{v} = \frac{\partial D_{x}}{\partial x} + \frac{\partial D_{y}}{\partial y} + \frac{\partial D_{z}}{\partial z}$$

$$\rho_{v} = \frac{\partial}{\partial x} (2y^{2} + z) + \frac{\partial}{\partial y} (4xy) + \frac{\partial}{\partial z} (x)$$

$$\rho_{v} = 0 + 4x + 0 = 4x$$

$$\rho_{v} at (-1,0,3) \text{ is given by}$$

$$\rho_{v} = 4(-1) = -4 C/m^{3}$$

$$Q = \int \rho_{v} dv = \int 4x \, dx \, dy \, dz$$

$$Q = \int 0^{1} 4x \, dx \, \int 0^{1} dy \, \int 0^{1} dz = \left[\frac{4x^{2}}{2}\right]_{0}^{1} [y]_{0}^{1} [z]_{0}^{1} = 2C$$
But from Course's law

But from Gauss's law

$$\psi = Q = 2 C$$

(c) The total charge enclosed by the cube is given by
 $Q = \psi = 2 C$

Q = ψ = 2 C 7. Given that D = Z ρ cos²φ a_z C/m², Calculate the charge density at (1, π/4,3) and the total charge enclosed by the cylinder of radius 1 m with -2 ≤ Z ≤ 2 m.

Answer:

Given that

$$D = Z \rho \cos^2 \varphi a_z C/m^2$$
$$\rho_v = \nabla \cdot D$$
vstem

Express in Cartesian coordinate system

$$\rho_{v} = \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho D_{\rho}) + \frac{\partial}{\partial \varphi} (D_{\varphi}) + \frac{\partial}{\partial z} (\rho D_{z}) \right]$$

$$\rho_{v} = \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho(0)) + \frac{\partial}{\partial \varphi} (0) + \frac{\partial}{\partial z} (\rho Z \rho \cos^{2} \varphi) \right]$$

$$\rho_{v} = \frac{1}{\rho} \left[\frac{\partial}{\partial z} (\rho Z \rho \cos^{2} \varphi) \right] = \rho \cos^{2} \varphi$$

$$\rho_{v} at \left(1, \frac{\pi}{4}, 3 \right) is given by$$

$$\rho_{v} = (1) \cos^{2} \left(\frac{\pi}{4} \right) = 0.5 C/m^{3}$$

The total charge enclosed by the cylinder is obtained as follows:

$$Q = \int \rho_v dv = \int \rho \cos^2 \varphi \ \rho d\rho d\varphi dz$$
$$Q = \int_0^1 \rho^2 d\rho \ \int_0^{2\pi} \cos^2 \varphi \ d\varphi \ \int_{-2}^2 dz$$
$$Q = \frac{1}{3}(\pi)(4) = 4.188 \ C$$

8. A charge distribution with spherical symmetry has density

$$\rho_{v} = \begin{cases} \frac{\rho_{0} r}{R}, & 0 \le r \le R \\ 0, & r > R \end{cases}$$

Determine E every where Answer: Given that

$$\rho_{v} = \begin{cases} \frac{\rho_{0} r}{R}, & 0 \le r \le R\\ 0, & r > R \end{cases}$$

(a) E for $r \leq R$, From Gauss's law

$$Q = \psi - 1$$

$$Q = \int \rho_v dv = \int \frac{\rho_0 r}{R} r^2 \sin \theta \, dr d\theta d\varphi$$

$$Q = \frac{\rho_0}{R} \int_0^r r^3 dr \int_0^\pi \sin \theta \, d\theta \int_0^{2\pi} d\varphi$$

$$Q = \frac{\rho_0}{R} \left(\frac{r^4}{4}\right) (2)(2\pi) = \frac{\rho_0}{R} \pi r^4 - 2$$

$$\psi = \int D \cdot ds = \int \varepsilon E \cdot ds = \varepsilon \int E_r a_r \cdot r^2 \sin \theta \, d\theta d\varphi \, a_r$$

$$\psi = \varepsilon_0 \int E_r r^2 \sin \theta \, d\theta d\varphi = \varepsilon_0 E_r r^2 \int_0^\pi \sin \theta \, d\theta \int_0^{2\pi} d\varphi$$

$$\psi = \varepsilon_0 E_r r^2 (2)(2\pi) = 4\pi r^2 \varepsilon_0 E_r - 3$$

Substitute equations 2 and 3 in equation 1

$$4\pi r^{2}\varepsilon_{0}E_{r} = \frac{\rho_{0}}{R}\pi r^{4}$$
$$E = \frac{\rho_{0}r^{2}}{4\varepsilon_{0}R}a_{r}$$

(b) E for **r** > **R**, From Gauss's law

$$Q = \psi - 1$$

$$Q = \int \rho_v dv = \int_0^R \rho_v dv + \int_R^r \rho_v dv = \int_0^R \frac{\rho_0 r}{R} dv + 0 = \int \frac{\rho_0 r}{R} r^2 \sin \theta \, dr \, d\theta \, d\varphi$$

$$Q = \frac{\rho_0}{R} \int_0^R r^3 dr \int_0^\pi \sin \theta \, d\theta \int_0^{2\pi} d\varphi$$

$$Q = \frac{\rho_0}{R} \left(\frac{R^4}{4}\right) (2)(2\pi) = \pi \rho_0 R^3 - 2$$

$$\psi = \int D \cdot ds = \int \varepsilon E \cdot ds = \varepsilon \int E_r a_r \cdot r^2 \sin \theta \, d\theta \, d\varphi \, a_r$$

$$\psi = \varepsilon_0 \int E_r r^2 \sin \theta \, d\theta \, d\varphi = \varepsilon_0 E_r r^2 \int_0^\pi \sin \theta \, d\theta \int_0^{2\pi} d\varphi$$

$$\psi = \varepsilon_0 E_r r^2 (2)(2\pi) = 4\pi r^2 \varepsilon_0 E_r - 3$$
Substitute equations 2 and 3 in equation 1

$$4\pi r^2 \varepsilon_0 E_r = \pi \rho_0 R^3$$

$$\mathcal{E}_0 \mathcal{E}_r = \pi \rho_0 R^3$$
$$\mathcal{E} = \frac{\rho_0 R^3}{4\varepsilon_0 r^2} a_r$$

9. Define an electric dipole and derive the potential and electric field intensity due to electric dipole?

Answer:



Figure 1.30 An electric dipole

Whenever two point charges of equal amplitudes and opposite sign and separated by a small distance then it is known as electric dipole. Let us derive the potential and electric field intensity due to electric dipole shown in figure above.

The potential at point 'P' is given by

$$V = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] = \frac{Q}{4\pi\varepsilon_0} \left[\frac{r_2 - r_1}{r_1 r_2} \right] - 1$$

Where r_1 and r_2 are the distances in between P and +Q, P and –Q respectively as shown in figure above. From figure,

$$r_{2} - r_{1} = d \cos \theta \qquad -2$$

$$r_{1}r_{2} \cong r^{2} \qquad -3$$
Substitute equations 2 and 3 in equation 1
$$V = \frac{Q}{4\pi\varepsilon_{0}} \left[\frac{d \cos \theta}{r^{2}} \right] \qquad -4$$

But

$$d \cdot a_r = d |a_r| \cos \theta = d \cos \theta - 5$$

Substitute equation 5 in equation 4

$$V = \frac{Qd \cdot a_r}{4\pi\varepsilon_0 r^2} = \frac{P \cdot a_r}{4\pi\varepsilon_0 r^2} \qquad -6$$

Where P = Qd is known as dipole moment. The dipole moment is defined as the product of the charge(Q) and distance between the charges(d). The direction of dipole moment is always from negative charge towards positive charge.

We know the relation between V and E as

 $E = -\nabla V$

Express above equation in spherical coordinate system

$$E = -\left[\frac{\partial V}{\partial r}a_r + \frac{1}{r}\frac{\partial V}{\partial \theta}a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}a_\varphi\right]$$

In case of practical dipole(antenna), the electric fields vary w.r.t 'r' and ' θ ' only. Hence Ø-components will be neglected.

$$E = -\left[\frac{\partial V}{\partial r}a_r + \frac{1}{r}\frac{\partial V}{\partial \theta}a_\theta\right] = -\frac{\partial V}{\partial r}a_r - \frac{1}{r}\frac{\partial V}{\partial \theta}a_\theta \qquad -7$$

Substitute equation 4 in equation 7

$$E = -\frac{\partial}{\partial r} \left(\frac{Qd \cos \theta}{4\pi\varepsilon_0 r^2} \right) a_r - \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{Qd \cos \theta}{4\pi\varepsilon_0 r^2} \right) a_\theta$$
$$E = -\left(\frac{Qd \cos \theta (-2)}{4\pi\varepsilon_0 r^3} \right) a_r - \frac{1}{r} \left(\frac{-Qd \sin \theta}{4\pi\varepsilon_0 r^2} \right) a_\theta$$
$$E = \frac{Qd \cos \theta}{2\pi\varepsilon_0 r^3} a_r + \frac{Qd \sin \theta}{4\pi\varepsilon_0 r^3} a_\theta$$

10. Derive the continuity equation and relaxation time? Answer:

The continuity equation states that, the rate of decrease of charge within a given volume must be equal to the net outward current flow through the closed surface of the volume. That is

$$I = -\frac{dQ_{in}}{dt} - 1$$

We know that,

$$I = \int J \cdot ds \qquad -2$$
$$Q = Q_{in} = \int \rho_v dv \qquad -3$$

Substitute equations 2 and 3 in equation 1

$$\int J \cdot ds = -\frac{d}{dt} \int \rho_v dv = -\int \frac{\partial \rho_v}{\partial t} dv \qquad -4$$

By using divergence theorem

$$\int J \cdot ds = \int (\nabla \cdot J) dv \qquad -5$$

Substitute equation 5 in equation4

$$\int (\nabla \cdot J) dv = -\int \frac{\partial \rho_v}{\partial t} dv$$
$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} - 6$$

The above equation is known as continuity equation in point form.

The relaxation time is defined as the time require for a charge placed in the interior of the material to drop e^{-1} or 37% of its initial value. The equation for relaxation time is derive as follows: We know that,

$$J = \sigma E - 7$$

Substitute equation 7 in equation 6

$$\nabla \cdot \sigma E = -\frac{\partial \rho_v}{\partial t} \qquad -8$$

But

$$\rho_{v} = \nabla \cdot D = \varepsilon \big(\nabla \cdot E \big)$$
$$\nabla \cdot E = \frac{\rho_{v}}{\varepsilon}$$

Multiply on both sides with σ

$$\nabla \cdot \sigma E = \sigma \frac{\rho_v}{\varepsilon} \qquad -9$$

 $-\frac{\partial \rho_v}{\partial t} = \sigma \frac{\rho_v}{\varepsilon}$

Equate equations 8 and 9

$$\frac{1}{\rho_v}\partial\rho_v = \frac{\sigma}{\varepsilon}\partial t$$

Take integration on both sides

$$\int \frac{1}{\rho_v} \partial \rho_v = \int \frac{\sigma}{\varepsilon} \partial t$$
$$\ln \rho_v = -\frac{\sigma}{\varepsilon} t + A \qquad -10$$

At $t = 0, \rho_v = \rho_{v0}$

Apply above initial condition to equation 10

$$\ln \rho_{\nu 0} = -\frac{\sigma}{\varepsilon}(0) + A$$
$$A = \ln \rho_{\nu 0} - 11$$

Substitute equation 11 in equation 10

$$\ln \rho_{v} = -\frac{\sigma}{\varepsilon}t + \ln \rho_{v0}$$
$$\ln \rho_{v} - \ln \rho_{v0} = -\frac{\sigma}{\varepsilon}t$$
$$\ln \left(\frac{\rho_{v}}{\rho_{v0}}\right) = -\frac{\sigma}{\varepsilon}t$$
$$\frac{\rho_{v}}{\rho_{v0}} = e^{-\frac{\sigma}{\varepsilon}t}$$
$$= \rho_{v0}e^{-\frac{\sigma}{\varepsilon}t} = \rho_{v0}e^{-t/T_{r}} - 12$$

Where

$$T_r = \frac{\varepsilon}{\sigma}$$
 is known as relaxation time

- 11. Given the potential $V = \frac{10}{r^2} \sin \theta \cos \varphi$ (a) Find the electric flux density D at $(2, \pi/2, 0)$

 $V=\frac{10}{r^2}\sin\theta\cos\varphi$

 ho_v

(b) Calculate the work done in moving a 10 µC charge from point A(1, 30⁰, 120⁰) to B(4, 90⁰, **60**⁰).

Answer:

(a)

Given

$$D = \varepsilon E$$

. . . .

....

$$E = -\nabla V = -\left(\frac{\partial V}{\partial r}a_r + \frac{1}{r}\frac{\partial V}{\partial \theta}a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}a_\varphi\right)$$
$$E = -\left(\frac{\partial}{\partial r}\left(\frac{\mathbf{10}}{r^2}\sin\theta\cos\varphi\right)a_r + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\mathbf{10}}{r^2}\sin\theta\cos\varphi\right)a_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\left(\frac{\mathbf{10}}{r^2}\sin\theta\cos\varphi\right)a_\varphi\right)$$

$$E = -\left(-\frac{20\sin\theta\cos\varphi}{r^3}a_r + \frac{10\cos\theta\cos\varphi}{r^3}a_\theta - \frac{10\sin\varphi}{r^3}a_\varphi\right)$$
$$E = \frac{20\sin\theta\cos\varphi}{r^3}a_r - \frac{10\cos\theta\cos\varphi}{r^3}a_\theta + \frac{10\sin\varphi}{r^3}a_\varphi$$
$$D = \varepsilon_0\left(\frac{20\sin\theta\cos\varphi}{r^3}a_r - \frac{10\cos\theta\cos\varphi}{r^3}a_\theta + \frac{10\sin\varphi}{r^3}a_\varphi\right)$$
$$(2, \pi/2, 0) \text{ is given by}$$

D at $(2, \pi/2, 0)$ is given

$$D = 8.854 \times 10^{-12} \left(\frac{20\sin(\pi/2)\cos(0)}{2^3} a_r - \frac{10\cos(\pi/2)\cos(0)}{2^3} a_\theta + \frac{10\sin(0)}{2^3} a_\varphi \right)$$
$$D = 22.1 a_r \ pC/m^2$$

(b) Work done is given by

$$W = QV_{AB} = Q(V_B - V_A)$$

$$V_A = V \text{ at } \mathbf{A} (\mathbf{1}, \mathbf{30^0}, \mathbf{120^0}) = V = \frac{\mathbf{10}}{\mathbf{1^2}} \sin(\mathbf{30}) \cos(\mathbf{120}) = -2.5$$
$$V_B = V \text{ at } \mathbf{B} (\mathbf{4}, \mathbf{90^0}, \mathbf{60^0}) = V = \frac{\mathbf{10}}{\mathbf{4^2}} \sin(\mathbf{90}) \cos(\mathbf{60}) = 0.3125 V$$
$$W = \mathbf{10} \times \mathbf{10^{-6}} (0.3125 - (-2.5)) = 28.125 \,\mu J$$

12. If V = x - y + xy + 2z V, find E at (1, 2, 3) and electrostatic energy stored in a cube of side 2 m centered at the origin

Answer:

Given

$$V = x - y + xy + 2z$$
 Volts

$$E = -\nabla V = -\left(\frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z\right)$$

$$E = -\left(\frac{\partial}{\partial x}(x - y + xy + 2z)a_x + \frac{\partial}{\partial y}(x - y + xy + 2z)a_y + \frac{\partial}{\partial z}(x - y + xy + 2z)a_z\right)$$

$$E = -\left((1 + y)a_x + (-1 + x)a_y + 2a_z\right)$$

$$E = -\left(1 + y)a_x - (-1 + x)a_y - 2a_z\right)$$

E at (1, 2, 3) is given by

$$E = -(1+2)a_x - (-1+1)a_y - 2a_z = -3a_x - 2a_z \quad V/m$$

s given by

The energy is given by

$$W = \frac{1}{2}\varepsilon_0 \int |E|^2 dv$$

$$|E| = \sqrt{3^2 + 2^2} = 3.6$$

$$W = \frac{1}{2} (8.854 \times 10^{-12}) \int (3.6)^2 dx dy dz = \frac{1}{2} (8.854 \times 10^{-12}) (3.6)^2 \int_0^2 dx \int_0^2 dy \int_0^2 dz$$

$$W = 0.46 n J$$

UNIT-II (MAGNETOSTATICS)

1. State and explain Biot-Savart Law? Answer:



Biot-Savart law states that, the magnetic field intensity 'dH' produced by the differential current element 'Idl' is proportional to the product 'Idl" and sine of the angle ' α ' between the element and the line joining the point 'P' to the element and is inversely proportional to the square of the distance 'R' between the element and Point 'P'. Biot-Savart law is also known as Ampere's law for current element.

$$dH = \propto \frac{Idl \sin \alpha}{R^2}$$
$$dH = k \frac{Idl \sin \alpha}{R^2}$$

Where k is known as proportionality constant which is equal to $1/4\pi$ to satisfy the system of units.

$$dH = \frac{Idl\,\sin\alpha}{4\pi R^2}$$

But

$$Idl \sin \alpha = Idl |a_R| \sin \alpha = Idl \times a_R$$

$$dH = \frac{Idl \times a_R}{4\pi R^2}$$

Take integration on both sides

$$\int dH = \int \frac{Idl \times a_R}{4\pi R^2}$$
$$H = \int \frac{Idl \times a_R}{4\pi R^2}$$

The above equation is known as general equation for the magnetic field intensity due to line current. Similarly the magnetic field intensity due to sheet current is given by

$$H = \int \frac{Kds \times a_R}{4\pi R^2}$$

Where 'K' is known as sheet current density in $ampers/m^2$ Similarly the magnetic field intensity due to volume current is given by

$$H = \int \frac{Jd\nu \times a_R}{4\pi R^2}$$

Where 'J' is known as volume current density in ampers/m³. The direction of H can be find by using either right hand thumb rule or right hand screw rule.

2. State and explain Ampere's Circuital law?

Answer:

Ampere's circuital law states that, the line integral of magnetic field intensity(H) around any closed path is equal to the net current (I_{enc}) enclosed by that path. That is

$$\int H \cdot dl = I_{enc} - 1$$

We have

$$I = I_{enc} = \int J \cdot ds \qquad -2$$

Substitute equation 2 in equation 1

$$\int H \cdot dl = \int J \cdot ds \qquad -3$$

The above equation is known as integral form of Ampere's circuital law. By using stokes theorem

$$\int_{\Omega} H \cdot dl = \int \nabla \times H \cdot ds \qquad -4$$

Substitute equation 4 in equation 3

$$\int \nabla \times H \cdot ds = \int J \cdot ds$$
$$\nabla \times H = J \qquad -5$$

The above equation is known as point form of Ampere's circuital law. Ampere's circuital alw is also known as Ampere's work law.

3. What are the applications of Ampere's circuital law and explain any two? Answer:

The applications of Ampere's circuital law are

- (i) Infinite line current
- (ii) Infinite sheet current
- (iii) Infinitely long coaxial transmission line

Infinite line current:



The statement of Ampere's circuital law is

$$\int H \cdot dl = I_{enc} - 1$$

$$\int H \cdot dl = \int H_{\varphi} a_{\varphi} \cdot (d\rho a_{\rho} + \rho d\varphi a_{\varphi} + dz a_{z})$$

$$\int H \cdot dl = \int H_{\varphi} (\rho d\varphi a_{\varphi}) = H_{\varphi} \rho \int_{0}^{2\pi} d\varphi a_{\varphi} = H_{\varphi} \rho (2\pi) a_{\varphi}$$

$$\int H \cdot dl = 2\pi \rho H_{\varphi} - 2$$

$$I_{enc} = I - 3$$

Substitute equations 2 and 3 in 1

$$2\pi\rho H_{\varphi} = I$$
$$H_{\varphi} = \frac{I}{2\pi\rho}$$
$$H = \frac{I}{2\pi\rho}a_{\varphi}$$

Infinite sheet of current:

The statement of Ampere's circuital law is

$$\int H \cdot dl = I_{enc} - 1$$

Before evaluating $\int H \cdot dl$ first let us find out the direction of H by using right hand thumb rule. H can be written as



$$H = \begin{cases} H_0 a_{x,} & \text{for } z > 0 \\ -H_0 a_{x,} & \text{for } z < 0 \end{cases} -2$$

$$\int H \cdot dl = \int_1^2 H \cdot dl + \int_2^3 H \cdot dl + \int_3^4 H \cdot dl + \int_4^1 H \cdot dl$$

Substitute equations 3 and 4 in equation 1

$$2H_0b = K_yb$$
$$H_0 = \frac{K_y}{2} - 5$$

Substitute equation 5 in equation 2

$$H = \begin{cases} \frac{K_y}{2} a_{x_i} & \text{for } z > 0\\ -\frac{K_y}{2} a_{x_i} & \text{for } z < 0 \end{cases}$$

In general, the H due to any current sheet can be written as

$$H = \frac{1}{2}K \times a_n$$

Where ' a_n ' represent the unit vector normal to the current sheet.

4. Derive the magnetic energy stored in magneto static field? Answer:

We know that, the equation for magnetic energy stored in an inductor is



The statement of Ampere's circuital law is

$$H \cdot dl = I_{enc} - 2$$

- 5

From above figure,

$$\int H \cdot dl = H \int dl = H \Delta y - 3$$

$$I_{anc} = \Delta I - 4$$

Substitute equations 3 and 4 in equation 2

 $\Delta I = H \Delta y$

We know that,

$$L = N \frac{\psi}{I} = \frac{\psi}{I} \qquad \text{when } N = 1$$
$$\Delta L = \frac{\Delta \psi}{\Delta I} \qquad -6$$

But

$$\psi = \int B \cdot ds$$
$$d\psi = B \cdot ds = \mu H.ds = \mu Hdxdz$$

Or

$$\Delta \psi = \mu H \Delta x \Delta z \qquad -7$$

Substitute equation 7 in equation 6

$$\Delta L = \frac{\mu H \Delta x \Delta z}{\Delta I} \qquad -8$$

Substitute equation 8 in equation1

$$\Delta W_m = \frac{1}{2} \left(\frac{\mu H \Delta x \Delta z}{\Delta I} \right) \Delta I^2 = \frac{1}{2} (\mu H \Delta x \Delta z) \Delta I \qquad -9$$

Substitute equation 5 in equation 9

$$\Delta W_m = \frac{1}{2} (\mu H \Delta x \Delta z) H \Delta y$$
$$\Delta W_m = \frac{1}{2} \mu H^2 \Delta x \Delta z \Delta y = \frac{1}{2} \mu H^2 \Delta v$$
$$dW_m = \frac{1}{2} \mu H^2 dv \qquad -10$$

Take integration on both sides

$$\int dW_m = \int \frac{1}{2} \mu H^2 dv$$
$$W_m = \int \frac{1}{2} \mu H^2 dv - 11$$

The above equation represents the energy stored in magneto static fields. But

$$B = \mu H$$

$$W_m = \int \frac{1}{2} \mu \left(\frac{B}{\mu}\right)^2 dv = \int \frac{B^2}{2\mu} dv \qquad -12$$

$$W_m = \int \frac{1}{2} \mu H H dv = \int \frac{1}{2} B H dv \qquad -13$$

The magnetic energy density (W_{md}) can be obtained from equation 10 as

$$W_{md} = \frac{dW_m}{dv} = \frac{1}{2}\mu H^2 = \frac{B^2}{2\mu} = \frac{1}{2}BH$$

5. Find out the magnetic field intensity due to infinite line current? Answer:



The general equation for magnetic field intensity due line current is given by

$$H = \int \frac{Idl \times a_R}{4\pi |R|^2} \qquad -1$$

From figure

$$dl = dza_z - 2$$

The distance vector 'R' is

$$R = \rho a_{\rho} - z a_z$$

$$|R| = \sqrt{\rho^2 + z^2} - 3$$

$$a_R = \frac{R}{|R|} = \frac{\rho a_{\rho} - z a_z}{\sqrt{\rho^2 + z^2}} - 4$$

Substitute equations 2, 3 and 4 in equation 1

$$H = \int_{-L/2}^{L/2} \frac{Idz a_z \times (\rho a_\rho - z a_z)}{4\pi \left| \sqrt{\rho^2 + z^2} \right|^2 (\sqrt{\rho^2 + z^2})}$$
$$H = \frac{I}{4\pi} \int_{-L/2}^{L/2} \frac{\rho dz}{(\rho^2 + z^2)^{3/2}} a_\varphi = \frac{I\rho}{4\pi} (2) \int_0^{L/2} \frac{1}{(\rho^2 + z^2)^{3/2}} dz a_\varphi$$
$$H = \frac{I\rho}{2\pi} \left[\frac{z}{\rho^2 (\sqrt{\rho^2 + z^2})} \right]_0^{L/2} a_\varphi = \frac{I\rho}{2\pi\rho^2} \left[\frac{L/2}{\sqrt{\rho^2 + (L/2)^2}} \right] a_\varphi$$
$$H = \frac{I}{2\pi\rho} \left[\frac{\frac{L}{2}}{\sqrt{\frac{4\rho^2 + L^2}{4}}} \right] a_\varphi = \frac{I}{2\pi\rho} \left[\frac{L}{\sqrt{4\rho^2 + L^2}} \right] a_\varphi - 5$$

The above equation represents the magnetic field intensity due to finite length (L) line current. For infinite length, length L is very large and hence $4\rho^2$ in above can be neglected as compared with L^2 .

$$H = \frac{I}{2\pi\rho} \left[\frac{L}{\sqrt{L^2}} \right] a_{\varphi} = \frac{I}{2\pi\rho} \left[\frac{L}{L} \right] a_{\varphi}$$
$$H = \frac{I}{2\pi\rho} a_{\varphi}$$

The above equation represents the magnetic field intensity due to infinite line current.

6. Find the magnetic field intensity due to infinitely long coaxial cable? Answer:

Consider the cross section of coaxial cable shown in figure below. By using Ampere's circuital law we need to find the magnetic field intensity.



There are four possible cases of amperian paths such as

(i)
$$0 \le \rho \le a$$

(ii) $a \le \rho \le b$
(iii) $b \le \rho \le (b+t)$
(iv) $\rho \ge (b+t)$
Case(i): $0 \le \rho \le a$
The statement of ampere's circuital law is
 $\int H \cdot dl = I_{enc} - 1$
 $\int H \cdot dl = \int H_{\varphi} a_{\varphi} \cdot (d\rho a_{\rho} + \rho d\varphi a_{\varphi} + dz a_{z}) = \int H_{\varphi}(\rho d\varphi)$
 $\int H \cdot dl = H_{\varphi} \rho \int_{0}^{2\pi} d\varphi = H_{\varphi} \rho(2\pi)$
 $\int H \cdot dl = 2\pi \rho H_{\varphi} - 2$
 $I_{enc} = \int J \cdot ds = \int \frac{1}{\pi a^{2}} \cdot ds = \int \frac{1}{\pi a^{2}} \rho d\rho d\varphi$
 $I_{enc} = \frac{1}{\pi a^{2}} \int_{0}^{\rho} \rho d\rho \int_{0}^{2\pi} d\varphi = \frac{1}{\pi a^{2}} \left(\frac{\rho^{2}}{2}\right)(2\pi)$
 $I_{enc} = \frac{I\rho^{2}}{a^{2}} - 3$
Substitute equations 2 and 3 in equation 1

$$2\pi\rho H_{\varphi} = \frac{I\rho^2}{a^2}$$
$$H = \frac{I\rho}{2\pi a^2} a_{\varphi}$$

Case(ii): $a \le \rho \le b$ The statement of Ampere's circuital law is

$$\int H \cdot dl = I_{enc} - 1$$

$$\int H \cdot dl = \int H_{\varphi} a_{\varphi} \cdot (d\rho a_{\rho} + \rho d\varphi a_{\varphi} + dz a_{z})$$

$$\int H \cdot dl = \int H_{\varphi} (\rho d\varphi a_{\varphi}) = H_{\varphi} \rho \int_{0}^{2\pi} d\varphi a_{\varphi} = H_{\varphi} \rho (2\pi) a_{\varphi}$$

$$\int H \cdot dl = 2\pi \rho H_{\varphi} - 2$$

$$I_{enc} = I - 3$$

Substitute equations 2 and 3 in 1

$$2\pi\rho H_{\varphi} = I$$
$$H_{\varphi} = \frac{I}{2\pi\rho}$$
$$H = \frac{I}{2\pi\rho}a_{\varphi}$$

Case(iii): $b \le \rho \le (b + t)$ The statement of ampere's circuital law is

$$\int H \cdot dl = I_{enc} - 1$$

$$\int H \cdot dl = \int H_{\varphi}a_{\varphi} \cdot (d\rho a_{\rho} + \rho d\varphi a_{\varphi} + dz a_{z}) = \int H_{\varphi}(\rho d\varphi)$$

$$\int H \cdot dl = H_{\varphi}\rho \int_{0}^{2\pi} d\varphi = H_{\varphi}\rho(2\pi)$$

$$\int H \cdot dl = 2\pi\rho H_{\varphi} - 2$$

$$I_{enc} = I + \int J \cdot ds = I + \int \frac{-I}{\pi[(b+t)^{2} - b^{2}]} \cdot ds$$

$$= I\left(1 - \int \frac{1}{\pi[b^{2} + t^{2} + 2bt - b^{2}]} \cdot \rho d\rho d\varphi\right)$$

$$I_{enc} = I\left(1 - \int \frac{1}{\pi(t^{2} + 2bt)} \int_{b}^{\rho} \rho d\rho \int_{0}^{2\pi} d\varphi\right)$$

$$I_{enc} = I\left(1 - \frac{1}{\pi(t^{2} + 2bt)} \left[\frac{\rho^{2}}{2}\right]_{b}^{\rho}(2\pi)\right)$$

$$I_{enc} = I\left(1 - \frac{\rho^{2} - b^{2}}{t^{2} + 2bt}\right) - 3$$

Substitute equations 2 and 3 in equation 1

$$2\pi\rho H_{\varphi} = I\left(1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right)$$
$$H = \frac{I}{2\pi\rho}\left(1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right)$$

 $Case(iv): \rho \ge (b+t)$

The statement of Ampere's circuital law is

$$\int H \cdot dl = I_{enc} - 1$$

$$\int H \cdot dl = \int H_{\varphi} a_{\varphi} \cdot (d\rho a_{\rho} + \rho d\varphi a_{\varphi} + dz a_{z})$$

$$\int H \cdot dl = \int H_{\varphi} (\rho d\varphi a_{\varphi}) = H_{\varphi} \rho \int_{0}^{2\pi} d\varphi a_{\varphi} = H_{\varphi} \rho (2\pi) a_{\varphi}$$

$$\int H \cdot dl = 2\pi \rho H_{\varphi} - 2$$

$$I_{enc} = I - I = 0 - 3$$

Substitute equations 2 and 3 in 1

$$2\pi\rho H_{\varphi} = 0$$
$$H = 0$$

7. Find out the magnetic field intensity due to infinitely long solenoid? Answer:

The cross section of solenoid is shown in figure below



The magnetic field intensity due to circular loop is given by

$$H = \frac{I\rho^2}{2(\rho^2 + h^2)^{\frac{3}{2}}} a_z$$
$$dH = \frac{Idl \rho^2}{2(\rho^2 + h^2)^{3/2}} a_z$$

From figure,

$$dl = ndz, \quad \rho = a \quad wirh \, n = N/l$$
$$dH = \frac{Indz \, a^2}{2(a^2 + h^2)^{3/2}} a_z \qquad -1$$

 $\cot \theta =$ $z = a \cot \theta$

 $\frac{z}{a}$

From Figure,

$$\frac{dz}{d\theta} = -a \csc^2 \theta$$
$$dz = -a \csc^2 \theta d\theta = -a \csc^3 \theta \sin \theta d\theta - 2$$

From figure,

$$cosec\theta = \frac{\sqrt{a^2 + z^2}}{a}$$
$$cosec^3\theta = \frac{\left(a^2 + z^2\right)^{\frac{3}{2}}}{a^3} - 3$$

Substitute equation 3 in equation 2

$$dz = -a \frac{(a^2 + z^2)^{\frac{3}{2}}}{a^3} \sin\theta \, d\theta = -\frac{(a^2 + z^2)^{\frac{3}{2}}}{a^2} \sin\theta \, d\theta \qquad -4$$

2

Substitute equation 4 in equation 1

$$dH = \frac{\ln a^2}{2(a^2 + h^2)^{3/2}} \left(-\frac{(a^2 + z^2)^2}{a^2} \sin\theta \ d\theta \right) a_z$$
$$dH = -\frac{nI}{2} \sin\theta \ d\theta \ a_z$$

Take integration on both sides

$$H = -\int \frac{nI}{2} \sin\theta \, d\theta \, a_z = -\frac{nI}{2} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta \, a_z$$
$$H = \frac{nI}{2} (\cos\theta_2 - \cos\theta_1) a_z \qquad -5$$

For infinite length, $\theta_2 = 0^0$ and $\theta_1 = 180^0$ Then

$$H = \frac{nI}{2} (\cos(0^0) - \cos(180^0)) a_z = \frac{nI}{2} (2) a_z$$
$$H = nI a_z$$

8. Find out the magnetic field intensity due to toroid? Answer:



In case of toroid, there are two possibilities of amperian paths such as inside i.e $(\rho_0 - a) \le \rho \le$ $(\rho_0 + a)$ and outside i.e. $\rho \ge (\rho_0 + a)$

Case(i) Inside:

The statement of Ampere's circuital law is

$$\int H \cdot dl = I_{enc} - 1$$

$$\int H \cdot dl = \int H_{\varphi} a_{\varphi} \cdot (d\rho a_{\rho} + \rho d\varphi a_{\varphi} + dz a_{z})$$

$$\int H \cdot dl = \int H_{\varphi} (\rho d\varphi a_{\varphi}) = H_{\varphi} \rho \int_{0}^{2\pi} d\varphi a_{\varphi} = H_{\varphi} \rho (2\pi) a_{\varphi}$$

$$\int H \cdot dl = 2\pi \rho H_{\varphi} - 2$$

$$I_{enc} = NI - 3$$

Substitute equations 2 and 3 in equation 1

$$2\pi\rho H_{\varphi} = NI$$
$$H = \frac{NI}{2\pi\rho}a_{\varphi}$$

Case(ii) Outside: The statement of Ampere's circuital law is

$$\int H \cdot dl = I_{enc} - 1$$

$$\int H \cdot dl = \int H_{\varphi} a_{\varphi} \cdot (d\rho a_{\rho} + \rho d\varphi a_{\varphi} + dz a_{z})$$

$$\int H \cdot dl = \int H_{\varphi} (\rho d\varphi a_{\varphi}) = H_{\varphi} \rho \int_{0}^{2\pi} d\varphi a_{\varphi} = H_{\varphi} \rho (2\pi) a_{\varphi}$$

$$\int H \cdot dl = 2\pi \rho H_{\varphi} - 2$$

$$I_{enc} = NI - NI = 0 - 3$$

Substitute equations 2 and 3 in equation 1

$$2\pi\rho H_{\varphi} = 0$$
$$H = 0$$

9. Determine the self inductance of a coaxial cable of inner radius 'a' and outer radius 'b' Answer:

The energy stored in inductor is given by

$$W_m = \frac{1}{2}LI^2$$
$$L = \frac{2W_m}{I^2} - 1$$

The internal inductance of coaxial cable is obtained as follows:

We know that the equation for the magnetic field intensity with in the inner conductor is given by

$$H = \frac{I\rho}{2\pi a^2} a_{\varphi}$$

$$B = \mu H = \frac{\mu I \rho}{2\pi a^2} a_{\varphi}$$
$$|B| = \frac{\mu I \rho}{2\pi a^2} - 2$$

The equation for the energy stored in magneto static field is given by

$$W_m = \int \frac{|B|^2}{2\mu} dv \qquad -3$$

Substitute equation 2 in equation 3

$$W_m = \int \frac{\left(\frac{\mu I \rho}{2\pi a^2}\right)^2}{2\mu} dv = \int \frac{\mu^2 I^2 \rho^2}{4\pi^2 a^4} \frac{1}{2\mu} dv$$

But

$$dv = \rho d\rho d\varphi dz$$

$$W_{m} = \int \frac{\mu^{2} l^{2} \rho^{2}}{4\pi^{2} a^{4}} \frac{1}{2\mu} (\rho d\rho d\varphi dz) = \frac{\mu l^{2}}{8\pi^{2} a^{4}} \int_{0}^{a} \rho^{3} d\rho \int_{0}^{2\pi} d\varphi \int_{0}^{l} dz$$
$$W_{m} = \frac{\mu l^{2}}{8\pi^{2} a^{4}} \left(\frac{a^{4}}{4}\right) (2\pi)(l)$$
$$W_{m} = \frac{\mu l^{2} l}{16\pi} - 4$$

Substitute equation 4 in equation 1

$$L = \frac{2\left(\frac{\mu I^{2}l}{16\pi}\right)}{I^{2}} = \frac{2\mu I^{2}l}{16\pi I^{2}}$$
$$L = L_{in} = \frac{\mu l}{8\pi} - 5$$

Similarly the external inductance can be obtained as follows:

We know that the equation for the magnetic field intensity in between the inner and outer conductor of coaxial cable is given by

$$H = \frac{I}{2\pi\rho}a_{\varphi}$$
$$B = \mu H = \frac{\mu I}{2\pi\rho}a_{\varphi}$$
$$|B| = \frac{\mu I}{2\pi\rho} - 6$$

The equation for the energy stored in magneto static field is given by

$$W_m = \int \frac{|B|^2}{2\mu} dv \qquad -7$$

Substitute equation 6 in equation 7

$$W_m = \int \frac{\left(\frac{\mu I}{2\pi\rho}\right)^2}{2\mu} dv = \int \frac{\mu^2 I^2}{4\pi^2 \rho^2} \frac{1}{2\mu} dv$$

But

$$dv = \rho d\rho d\varphi dz$$

$$W_m = \int \frac{\mu^2 I^2}{4\pi^2 \rho^2} \frac{1}{2\mu} (\rho d\rho d\varphi dz) = \frac{\mu I^2}{8\pi^2} \int_0^a \frac{1}{\rho} d\rho \int_0^{2\pi} d\varphi \int_0^l dz$$

$$W_m = \frac{\mu l^2}{8\pi^2} (\ln (b/a))(2\pi)(l)$$
$$W_m = \frac{\mu l^2 l}{4\pi} \ln (b/a) - 8$$

Substitute equation 8 in equation 1

$$L = \frac{2\left(\frac{\mu l^2 l}{4\pi} \ln{(b/a)}\right)}{l^2} = \frac{2\mu l^2 l}{4\pi l^2} \ln{(b/a)}$$
$$L = L_{ext} = \frac{\mu l}{2\pi} \ln{(b/a)} - 9$$

Total inductance is given by

$$L = L_{in} + L_{ext} - 10$$

Substitute equations 5 and 9 in equation 10

$$L = \frac{\mu l}{8\pi} + \frac{\mu l}{2\pi} \ln(b/a) = \frac{\mu l}{2\pi} \left(\frac{1}{4} + \ln(b/a) \right)$$

10. A circular loop located on $x^2+y^2 = 9$, Z = 0 carriers a direct current of 10 A along a φ . Determine H at (0,0,4) and (0,0,-4).

Answer:



The general equation for magnetic field intensity due line current is given by

$$H = \int \frac{Idl \times a_R}{4\pi |R|^2} = \int \frac{Idl \times R}{4\pi |R|^3} \qquad -1$$

From figure

$$dl = \rho d\varphi a_{\varphi} - 2$$

The distance vector 'R' is

$$R = ha_z - \rho a_\rho$$
$$|R| = \sqrt{h^2 + \rho^2} \qquad -3$$

$$H = \int \frac{I\rho d\varphi a_{\varphi} \times (ha_{z} - \rho a_{\rho})}{4\pi \sqrt{h^{2} + \rho^{2}}^{3}} = \int \frac{I\rho (d\varphi a_{\varphi} \times ha_{z} - d\varphi a_{\varphi} \times \rho a_{\rho})}{4\pi (h^{2} + \rho^{2})^{3/2}}$$
$$H = \int \frac{I\rho (hd\varphi a_{\rho} - \rho d\varphi (-a_{z}))}{4\pi (h^{2} + \rho^{2})^{3/2}} = \int \frac{I\rho (hd\varphi a_{\rho} + \rho d\varphi a_{z})}{4\pi (h^{2} + \rho^{2})^{3/2}}$$

Due to symmetry of circular loop the p-components will be cancelled

$$H = \int \frac{I\rho(\rho d\varphi a_z)}{4\pi (h^2 + \rho^2)^{3/2}} a_z = \frac{I\rho^2}{4\pi (h^2 + \rho^2)^{3/2}} \int_0^{2\pi} d\varphi a_z$$
$$H = \frac{I\rho^2}{4\pi (h^2 + \rho^2)^{3/2}} (2\pi) a_z = \frac{I\rho^2}{2(h^2 + \rho^2)^{3/2}} a_z$$

H at (0,0,4) is given by

$$H = \frac{10X3^2}{2(4^2 + 3^2)^{3/2}} a_z = 0.36 \ a_z \ A/m$$

H at (0,0,-4) is given by

$$H = \frac{10X3^2}{2((-4)^2 + 3^2)^{3/2}} a_z = 0.36 a_z A/m$$

11. State and explain Faraday's law?

Answer:

Faraday's law states that, time varying magnetic field is able to produce an e.m.f or voltage with in a closed circuit. Consider the closed circuit placed nearby to the current carrying conductor as shown in figure below.



$$e.m.f = -N\frac{d\psi}{dt}$$

Where N is number of turns, ψ is magnetic flux. When N = 1, then

$$e.m.f = V = -\frac{d\psi}{dt} - 1$$

The relation between V and E is given by

$$V = \int E \cdot dl \qquad -2$$

Equate equations 1 and 2

$$\int E \cdot dl = -\frac{d\psi}{dt}$$

But

$$\psi = \int B \cdot ds$$
$$\int E \cdot dl = -\frac{d}{dt} \int B \cdot ds = -\int \frac{\partial B}{\partial t} \cdot ds$$
$$\int E \cdot dl = -\int \frac{\partial B}{\partial t} \cdot ds - 3$$

The above equation is known as integral form of Faraday's law By using stokes theorem,

$$\int E \cdot dl = \int \nabla \times E \cdot ds \qquad -4$$

Substitute equation 4 in equation 3

$$\int \nabla \times E \cdot ds = -\int \frac{\partial B}{\partial t} \cdot ds$$
$$\nabla \times E = -\frac{\partial B}{\partial t} - 5$$

The above equation is known as point form of Faraday's law.

If the e.m.f is induced in a stationary closed circuit due to time varying magnetic field then it is known as statically induced e.m.f or transformer e.m.f. Similarly if the e.m.f induced in a moving closed circuit due to time varying magnetic field, then it is known as dynamically induced e.m.f or motional e.m.f.

12. What is the inconsistency of Ampere's circuital law and explain about displacement current density?

Answer:

The point form of Ampere's circuital law is given by

$$\nabla \times H = J$$

Take Divergence on both sides

Take Divergence on both sides

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J$$

But from standard vector results, for any vector the divergence of curl of any vector is equal to zero. i.e. $\nabla \cdot (\nabla \times H) = 0$

 $0 = \nabla \cdot J \qquad -1$

But from continuity equation, we know that,

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \qquad -2$$

Form the continuity equation, the divergence of current density $(\nabla \cdot J)$ must some finite value but it should not be zero. Therefore equation 1 is not satisfying the continuity equation and hence something is missing in the ampere's circuital law. That means, along with current density (J), there must be some other parameter.

Let

$$\nabla \times H = J + G - 3$$
$$\nabla \cdot (\nabla \times H) = \nabla \cdot J + \nabla \cdot G$$

But from standard vector results, for any vector the divergence of curl of any vector ie equal to zero. i.e. $\nabla \cdot (\nabla \times H) = 0$

$$0 = \nabla \cdot J + \nabla \cdot G$$
$$\nabla \cdot G = -\nabla \cdot J - 4$$

Substitute equation 2 in equation 4

$$\nabla \cdot G = \frac{\partial \rho_{\nu}}{\partial t} \qquad -5$$

 $\rho_v = \nabla \cdot D - 6$

But

Substitute equation 6 in equation 5

$$\nabla \cdot G = \frac{\partial (\nabla \cdot D)}{\partial t} = \nabla \cdot \frac{\partial D}{\partial t}$$
$$G = J_d = \frac{\partial D}{\partial t} - 7$$

The above equation represents the displacement current density. The displacement current density is defined as the rate of change of electric displacement or electric flux density. Substitute equation 7 in equation 3

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

13. Write the Maxwell's equations in different final form and give their word statement? Answer:

Point form:

$$\nabla \cdot B = 0 \quad -1$$
$$\nabla \cdot D = \rho_v \quad -2$$
$$\nabla \times E = -\frac{\partial B}{\partial t} \quad -3$$
$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad -4$$
$$\int B \cdot ds = 0 \quad -1$$
$$\int D. \, ds = \int \rho_v \, dv \quad -2$$
$$\int E \cdot dl = -\int \frac{\partial B}{\partial t} \cdot ds \quad -3$$
$$\int H \cdot dl = \int \left(I + \frac{\partial D}{\partial t}\right) \cdot ds \quad -4$$

Integral form:

$$\int D \, ds = 0 \qquad 1$$

$$\int D \, ds = \int \rho_v \, dv \qquad -2$$

$$\int E \cdot dl = -\int \frac{\partial B}{\partial t} \cdot ds \qquad -3$$

$$\int H \cdot dl = \int \left(J + \frac{\partial D}{\partial t}\right) \cdot ds \qquad -$$

Word statement:

- The first Maxwell's equation represents the net magnetic flux emerging through any (i) closed surface is zero.
- The second Maxwell's equation represents that, the total electric flux density or total (ii) electric displacement through the surface enclosing a volume 'v' is equal to the total charge within the volume.

- (iii) The third Maxwell's equation represents that, the electric field intensity around any closed path is equal to the negative of the time derivative of the magnetic flux density through that closed surface.
- (iv) The fourth Maxwell's equation represents that, the magnetic field intensity around any closed path is equal to surface integration of conduction current density plus displacement current density.

Maxwell's equation in phasor form:

$$\nabla \cdot B = 0 \qquad -1$$

$$\nabla \cdot D = \rho_v \qquad -2$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -j\omega\mu H \qquad -3$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + j\omega\varepsilon E = (\sigma + j\omega\varepsilon)E \qquad -4$$

Maxwell's equation for free space:

For free space, $\sigma = 0$, $\rho_v = 0$

$$\nabla \cdot B = 0 \qquad -1$$
$$\nabla \cdot D = 0 \qquad -2$$
$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad -3$$
$$\nabla \times H = \frac{\partial D}{\partial t} = \frac{\partial D}{\partial t} \qquad -4$$

UNIT-III

1. State and explain the electric boundary conditions at dielectric-dielectric boundary interface?

Answer:

The four boundary conditions at the dielectric-dielectric boundary interface are given by

- (i) The tangential components of electric field intensity are continuous across the dielectric-dielectric boundary interface. i.e $E_{1t} = E_{2t}$
- (ii) The tangential components of electric flux density are discontinuous across the dielectric-dielectric boundary interface. i.e $D_{1t} \neq D_{2t}$
- (iii) The normal components of electric flux density are continuous across the dielectricdielectric boundary interface. i.e $D_{1n} = D_{2n}$
- (iv) The normal components of electric field intensity are discontinuous across the dielectric-dielectric boundary interface. i.e $E_{1n} \neq E_{2n}$

To prove the first two boundary conditions apply $\int E \cdot dl = 0$ to the closed path 'abcda' shown in figure below.



$$\int E \cdot dl = \int_{a}^{b} E \cdot dl + \int_{b}^{c} E \cdot dl + \int_{c}^{d} E \cdot dl + \int_{d}^{a} E \cdot dl = 0$$

$$E_{1t} \Delta w - E_{1n} \frac{\Delta h}{2} - E_{2n} \frac{\Delta h}{2} - E_{2t} \Delta w + E_{2n} \frac{\Delta h}{2} + E_{1n} \frac{\Delta h}{2} = 0$$

$$E_{1t} \Delta w - E_{2t} \Delta w = 0$$

$$(E_{1t} - E_{2t}) \Delta w = 0$$

$$E_{1t} - E_{2t} = 0$$

$$E_{1t} = E_{2t} - 1$$

We know that,

$$D = \varepsilon E$$

$$D_{1t} = \varepsilon_1 E_{1t}$$

$$E_{1t} = \frac{D_{1t}}{\varepsilon_1} - 2$$

$$E_{2t} = \frac{D_{2t}}{\varepsilon_2} - 3$$

Substitute equations 2 and 3 in equation 1

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$
$$D_{1t} = \frac{\varepsilon_1}{\varepsilon_2} D_{2t} - 4$$

Therefore

 $D_{1t} \neq D_{2t}$ becuase $\varepsilon_1 \neq \varepsilon_2$ Similarly to prove third and fourth boundary conditions apply Gauss's law to the pill box shown in figure below.



Region 2, Dielectric, ϵ_2

The statement of Gauss's law is

$$\psi = Q_{enc} - 5$$

$$\psi = \int D \cdot ds = D_{1n} \Delta S - D_{2n} \Delta S - 6$$

$$Q_{enc} = \rho_S \Delta S - 7$$

Substitute equations 6 and 7 in equation 5

$$D_{1n}\Delta S - D_{2n}\Delta S = \rho_S \Delta S$$

$$(D_{1n} - D_{2n})\Delta S = \rho_S \Delta S$$

$$D_{1n} - D_{2n} = \rho_S$$

In case of dielectrics $\rho_{\rm S} = 0$

$$D_{1n} - D_{2n} = 0 D_{1n} = D_{2n} - 8$$

We know that,

$$D = \varepsilon E$$

$$D_{1n} = \varepsilon_1 E_{1n} - 9$$

$$D_{2n} = \varepsilon_2 E_{2n} - 10$$

Substitute equations 9 and 10 in equation 8

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$
$$E_{1n} = \frac{\varepsilon_2}{\varepsilon_1} E_{2n} - 11$$

Therefore

$$E_{1n} \neq E_{2n}$$
 because $\varepsilon_1 \neq \varepsilon_2$

2. State and explain the electric boundary conditions at dielectric-conductor boundary interface?

Answer:

The four boundary conditions at the dielectric-conductor boundary interface are given by

- The tangential components of electric field intensity are continuous across the (v) dielectric-conductor boundary interface. i.e $E_{1t} = E_{2t}$
- The tangential components of electric flux density are continuous across the (vi) dielectric-conductor boundary interface. i.e $D_{1t} = D_{2t}$
- The normal components of electric flux density are discontinuous across the (vii) dielectric-conductor boundary interface. i.e $D_{1n} \neq D_{2n}$ The normal components of electric field intensity are discontinuous across the
- (viii) dielectric-conductor boundary interface. i.e $E_{1n} \neq E_{2n}$

To prove the first two boundary conditions apply $\int E \cdot dl = 0$ to the closed path 'abcda' shown in figure below.





$$\begin{aligned} \int E \cdot dl &= \int_{a}^{b} E \cdot dl + \int_{b}^{c} E \cdot dl + \int_{c}^{d} E \cdot dl + \int_{d}^{a} E \cdot dl = 0 \\ E_{1t} \Delta w - E_{1n} \frac{\Delta h}{2} - (0) \frac{\Delta h}{2} - (0) \Delta w + (0) \frac{\Delta h}{2} + E_{1n} \frac{\Delta h}{2} = 0 \\ E_{1t} \Delta w &= 0 \\ E_{1t} = 0 \end{aligned}$$

But $E_{2t} = 0$ because region 2 is good conductor
Therefore $E_{1t} = E_{2t} - 1$
We know that,

_

We know that, $D = \varepsilon E$ $D_{1t} = \varepsilon_1 E_{1t} = 0$ But $D_{2t} = 0$ because region 2 is good conductor Therefore $D_{1t} = D_{2t} - 2$ Similarly to prove third and fourth boundary conditions apply Gauss's law to the pill box shown in figure below



The statement of Gauss's law is

$$\psi = Q_{enc} - 3$$

$$\psi = \int D \cdot ds = D_{1n}\Delta S - (0)\Delta S = D_{1n}\Delta S - 4$$

 $Q_{enc} = \rho_S \, \Delta S$ - 5 Substitute equations 4 and 5 in equation 3 $D_{1n}\Delta S = \rho_S \, \Delta S$ $D_{1n} = \rho_S$ - 6 $D_{2n} = 0$ becuase region 2 is good conductor But Therefore $D_{1n} \neq D_{2n}$

We know that,

$$D = \varepsilon E$$
$$D_{1n} = \varepsilon_1 E_{1n} - 7$$

Substitute equations 7 in equation 6

$$\varepsilon_{1}E_{1n} = \rho_{S}$$

$$E_{1n} = \frac{\rho_{S}}{\varepsilon_{1}} - 8$$

$$E_{2n} = 0 \quad becuase \ region \ 2 \ is \ good \ conductor$$

$$E_{1n} \neq E_{2n}$$

But Therefore

3. State and explain the magnetic boundary conditions at dielectric-dielectric boundary interface?

Answer:

The four boundary conditions at the dielectric-dielectric boundary interface are given by

- The tangential components of magnetic field intensity are continuous across the (ix) dielectric-dielectric boundary interface. i.e $H_{1t} = H_{2t}$
- The tangential components of magnetic flux density are discontinuous across the (x) dielectric-dielectric boundary interface. i.e $B_{1t} \neq B_{2t}$
- (xi) The normal components of magnetic flux density are continuous across the dielectricdielectric boundary interface. i.e $B_{1n} = B_{2n}$
- The normal components of magnetic field intensity are discontinuous across the (xii) dielectric-dielectric boundary interface. i.e $H_{1n} \neq H_{2n}$

To prove the first two boundary conditions apply $\int H \cdot dl = I_{enc}$ to the closed path 'abcda' shown in figure below.





$$\int H \cdot dl = \int_{a}^{b} H \cdot dl + \int_{b}^{c} H \cdot dl + \int_{c}^{d} H \cdot dl + \int_{d}^{a} H \cdot dl = I_{enc}$$

$$H_{1t}\Delta w - H_{1n}\frac{\Delta h}{2} - H_{2n}\frac{\Delta h}{2} - H_{2t}\Delta w + H_{2n}\frac{\Delta h}{2} + H_{1n}\frac{\Delta h}{2} = k\Delta w$$

$$H_{1t}\Delta w - H_{2t}\Delta w = k\Delta w$$

$$(H_{1t} - H_{2t})\Delta w = k\Delta w$$

$$H_{1t} - H_{2t} = k$$

 $H_{1t} = H_{2t} \qquad -1$

But for dielectrics, k = 0

We know that,

$$B = \mu H$$

$$B_{1t} = \mu_1 H_{1t}$$

$$H_{1t} = \frac{B_{1t}}{\mu_1} - 2$$

$$H_{2t} = \frac{B_{2t}}{\mu_2} - 3$$

Substitute equations 2 and 3 in equation 1

$$\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$$
$$B_{1t} = \frac{\mu_1}{\mu_2} B_{2t} - 4$$

Therefore

$$B_{1t} \neq B_{2t}$$
 becuase $\mu_1 \neq \mu_2$

Similarly to prove third and fourth boundary conditions apply $\int B \cdot ds = 0$ to the pill box shown in figure below.



$$\int B \cdot ds = B_{1n}\Delta S - B_{2n}\Delta S = 0$$
$$B_{1n} - B_{2n} = 0$$
$$B_{1n} = B_{2n} - 5$$

We know that,

$$B = \mu H$$

$$B_{1n} = \mu_1 H_{1n} - 6$$

$$B_{2n} = \mu_2 H_{2n} - 7$$

ion 5

Substitute equations 6 and 7 in equation 5

$$\mu_1 H_{1n} = \mu_2 H_{2n}$$
$$H_{1n} = \frac{\mu_2}{\mu_1} H_{2n} - 8$$

Therefore

$$H_{1n} \neq H_{2n}$$
 because $\mu_1 \neq \mu_2$

4. State and explain the magnetic boundary conditions at dielectric-conductor boundary interface?

Answer:

The four boundary conditions at the dielectric-conductor boundary interface are given by

- (i) The tangential components of magnetic field intensity are discontinuous across the dielectric-conductor boundary interface. i.e $H_{1t} \neq H_{2t}$
- (ii) The tangential components of magnetic flux density are discontinuous across the dielectric-conductor boundary interface. i.e $B_{1t} \neq B_{2t}$
- (iii) The normal components of magnetic flux density are continuous across the dielectricconductor boundary interface. i.e $B_{1n} = B_{2n}$
- (iv) The normal components of magnetic field intensity are continuous across the dielectric-conductor boundary interface. i.e $H_{1n} = H_{2n}$

To prove the first two boundary conditions apply $\int H \cdot dl = I_{enc}$ to the closed path 'abcda' shown in figure below.

Region 1, Dielectric, μ_1





$$\int H \cdot dl = \int_{a}^{b} H \cdot dl + \int_{b}^{c} H \cdot dl + \int_{c}^{d} H \cdot dl + \int_{a}^{d} H \cdot dl = I_{enc}$$

$$H_{1t}\Delta w - H_{1n}\frac{\Delta h}{2} - (0)\frac{\Delta h}{2} - (0)\Delta w + (0)\frac{\Delta h}{2} + H_{1n}\frac{\Delta h}{2} = k\Delta w$$

$$H_{1t}\Delta w = k\Delta w$$

$$H_{1t} = k = -1$$
But
$$H_{2t} = 0$$
But
$$H_{2t} = 0$$
But
$$H_{2t} = 0$$
But
$$B_{1t} = \mu_{1}H_{1t}$$

$$H_{1t} = \frac{B_{1t}}{\mu_{1}} - 3$$
Substitute equation 3 in equation 1
$$\frac{B_{1t}}{B_{1t}} = k$$

$$B_{1t} = \mu_{1}k$$
But
$$B_{2t} = 0$$
But
$$B_{2t} = 0$$
But
$$B_{2t} = 0$$
But
$$B_{2t} = 0, B_{2t} = 0, B_{2n} = 0$$
Region 1, Dielectric, μ_{1}

$$Area = \Delta S$$
Pill Box
$$\int B \cdot ds = B_{1n}\Delta S - (0)\Delta S = 0$$

$$B_{1n} = B_{2n} - 5$$
But
Therefore
We know that,

 $B = \mu H$ $B_{1n} = \mu_1 H_{1n}$

$$H_{1n} = \frac{B_{1n}}{\mu_1} = 0$$

$$H_{2n} = 0 \qquad \text{because region 2 is good conductor}$$

efore
$$H_{1n} = H_{2n} \qquad -6$$

A parallel plate capacitor with plate area of 5 cm² and plate separation of

Therefore

But

5. A parallel plate capacitor with plate area of 5 cm² and plate separation of 3 mm has a voltage 50 sin10³t volts applied to its plates. Calculate the displacement current assuming $\varepsilon = 2\varepsilon_0$

Answer:

Given data:

Plate Area (A) = 5 cm² = 5 X 10⁻⁴ m²
Plate separation (d) = 3 mm
Voltage (V) = **50 sin10³t**

$$\varepsilon = 2\varepsilon_0, \varepsilon_r = 2$$

 $D = \varepsilon E = \varepsilon \frac{V}{d}$
 $J_d = \frac{\partial D}{\partial t} = \frac{\varepsilon}{d} \frac{\partial V}{\partial t}$
 $I_d = J_d \times A = \frac{\varepsilon A}{d} \frac{\partial V}{\partial t}$
 $I_d = \frac{2\varepsilon_0 A}{d} \frac{\partial V}{\partial t} = \frac{2 \times 8.854 \times 10^{-12} \times 5 \times 10^{-4}}{3 \times 10^{-3}} \frac{\partial}{\partial t} (50 \sin 10^3 t)$
 $I_d = 147.4 \cos 10^3 t nA$

- 6. In free space $E = 20 \cos (\omega t 50 x) a_y V/m$ Calculate
 - (a) J_d
 - **(b)** H
 - (c) w
- Answer:

Given

 $E = 20\cos\left(\omega t - 50\,x\right)a_y \qquad V/m$

In general

$$E = E_0 \cos{(\omega t - \beta x)a_y} \qquad V/m$$

Comparing above two equations

$$\beta = 50 \ rad/m$$

(a)

$$J_{d} = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = \varepsilon_{0} \frac{\partial}{\partial t} (20 \cos (\omega t - 50 x))$$

$$J_{d} = \varepsilon_{0} (-20 \sin (\omega t - 50 x). \omega)$$

$$J_{d} = -20 \omega \varepsilon_{0} \sin (\omega t - 50 x)$$

(b)

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} = -\mu \frac{dH}{dt}$$
$$H = -\frac{1}{\mu} \int (\nabla \times E) dt$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 20\cos(\omega t - 50x) & 0 \end{vmatrix}$$

$$\nabla \times E = a_x \left(\frac{\partial}{\partial y} (0) - \frac{\partial}{\partial z} (20\cos(\omega t - 50x)) \right) - a_y \left(\frac{\partial}{\partial x} (0) - \frac{\partial}{\partial z} (0) \right)$$

$$+ a_z \left(\frac{\partial}{\partial x} (20\cos(\omega t - 50x)) - \frac{\partial}{\partial y} (0) \right)$$

$$\nabla \times E = a_z \left(\frac{\partial}{\partial x} (20\cos(\omega t - 50x)) \right) = -20\sin(\omega t - 50x) \cdot (-50) a_z$$

$$\nabla \times E = 1000\sin(\omega t - 50x) a_z$$

$$H = -\frac{1}{\mu} \int (1000\sin(\omega t - 50x)) dta_z$$

$$H = -\frac{1}{\mu} (-1000\cos(\omega t - 50x)) \cdot \omega) a_z$$

$$H = \frac{1000\omega}{\mu_0} \cos(\omega t - 50x) a_z$$
we that

(c) We know that

$$\beta = -\frac{1}{c}$$
$$\omega = \beta c = 50 \times 3 \times 10^8 = 1.5 \times 10^{10} \text{ m/sec}$$

7. In a medium characterized by $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = 4\varepsilon_0$ and $E = 20 \sin (10^8 t - \beta z)a_y V/m$ Calculate β and H?

Answer:

In general

Given

$$\sigma = 0, \ \mu = \mu_0, \ \varepsilon = 4\varepsilon_0$$
$$E = 20 \sin(10^8 t - \beta z) a_y \ V/m$$
$$E = E_0 \sin(\omega t - \beta z) a_y \ V/m$$

By comparing above two equations

$$\omega = 10^{8} rad/sec$$

$$\beta = \frac{\omega}{v} = \frac{\omega}{\frac{1}{\sqrt{\mu\varepsilon}}} = \omega\sqrt{\mu\varepsilon} = 10^{8}\sqrt{\mu_{0}4\varepsilon_{0}} = 10^{8}\sqrt{4\pi \times 10^{-7} \times 4 \times 8.854 \times 10^{-12}}$$

$$\beta = 0.666 rad/m$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu\frac{\partial H}{\partial t} = -\mu\frac{dH}{dt}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} = -\mu \frac{\partial H}{\partial t}$$
$$H = -\frac{1}{\mu} \int (\nabla \times E) dt$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 20\sin(10^8 t - \beta z) & 0 \end{vmatrix}$$

$$\nabla \times E = a_x \left(\frac{\partial}{\partial y} (0) - \frac{\partial}{\partial z} (20\sin(10^8 t - \beta z)) \right) - a_y \left(\frac{\partial}{\partial x} (0) - \frac{\partial}{\partial z} (0) \right)$$

$$+ a_z \left(\frac{\partial}{\partial x} (20\sin(10^8 t - \beta z)) - \frac{\partial}{\partial y} (0) \right)$$

$$\nabla \times E = a_x \left(-\frac{\partial}{\partial z} (20\sin(10^8 t - \beta z)) \right) = -20\cos(10^8 t - \beta z).(-\beta) a_x$$

$$\nabla \times E = 20\beta\cos(10^8 t - \beta z) a_x$$

$$H = -\frac{1}{\mu} \int (20\beta\cos(10^8 t - \beta z)) dta_x$$

$$H = -\frac{1}{\mu \times 10^8} (20\beta\sin(10^8 t - \beta z)) a_x$$

$$H = -\frac{20\beta}{\mu_0 \times 10^8} \sin(10^8 t - \beta z) a_x$$

 $n = -\frac{1}{4\pi \times 10^{-7} \times 10^8} \sin(10^{\circ}t - \beta z)a_x = -0.106 \sin(10^{\circ}t - \beta z)a_x A/m$ 8. A medium is characterized by $\sigma = 0$, $\mu = 2\mu_0$, $\varepsilon = 5\varepsilon_0$. If $H = 2 \cos(\omega t - 3y)a_z A/m$ *m* Calculate ω and E?

Answer:

$$\sigma = 0, \mu = 2\mu_0, \epsilon = 5\epsilon_0$$

$$H = 2 \cos(\omega t - 3y)a_z A/m$$

$$H = H_0 \cos(\omega t - \beta y)a_z A/m$$

In general H = By comparing above two equations

$$\beta = 3 rad/m$$

$$\beta = \frac{\omega}{v}$$

$$\omega = \beta v = \beta \frac{1}{\sqrt{\mu \varepsilon}} = \frac{3}{\sqrt{2\mu_0 \times 5\varepsilon_0}} = \frac{3}{\sqrt{2 \times 4\pi \times 10^{-7} \times 5 \times 8.854 \times 10^{-12}}}$$

$$\omega = 2.846 \times 10^8 rad/sec$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \frac{\partial D}{\partial t} = \frac{\partial D}{\partial t}$$

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t} = \varepsilon \frac{dE}{dt}$$

$$E = \frac{1}{\varepsilon} \int (\nabla \times H) dt$$

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$
$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 2\cos(\omega t - 3y) \end{vmatrix}$$
$$\nabla \times H = a_x \left(\frac{\partial}{\partial y} (2\cos(\omega t - 3y)) - \frac{\partial}{\partial z} (0) \right) - a_y \left(\frac{\partial}{\partial x} (2\cos(\omega t - 3y)) - \frac{\partial}{\partial z} (0) \right)$$
$$+ a_z \left(\frac{\partial}{\partial x} (0) - \frac{\partial}{\partial y} (0) \right)$$
$$\nabla \times H = \left(\frac{\partial}{\partial y} (2\cos(\omega t - 3y)) \right) a_x$$
$$\nabla \times H = -2\sin(\omega t - 3y)(-3)a_x = 6\sin(\omega t - 3y)a_x$$
$$E = \frac{1}{\varepsilon} \int \left(6\sin(\omega t - 3y) \right) dt a_x = \frac{6}{5\varepsilon_0 \omega} (-\cos(\omega t - 3y))a_x$$
$$E = -\frac{6}{5 \times 8.854 \times 10^{-12} \times 2.846 \times 10^8} \cos(\omega t - 3y) a_x$$

 $E = -476.8 \cos(2.486 \times 10^8 t - 3y) a_x V/m$

9. Define polarization and explain types of polarization

Polarization refers to time varying behavior of the electric field strength vector 'E' at any point in a free space. For example a uniform plane wave traveling in the z-direction with the E and H vectors lying on the xy-plane. The electric field will have both E_x and E_y when the wave propagating along z-direction. When $E_y = 0$, then only E_x is present and the wave is said to polarized in the x-direction. Similarly when $E_x = 0$ and only E_y is present, then the wave is said to be polarized in the y-direction. If both E_x and E_y present then the direction of resultant vector 'E' depends upon the magnitude of both E_x and E_y . The angle made by 'E' with E_x is obtained from the following figure



Figure: Resultant electric vector 'E'

Basically there are three types of polarizations which are given by

- (i) Linear polarization
- (ii) Elliptical polarization

(iii) Circular polarization

Linear Polarization:

When the two components $(E_x \text{ and } E_y)$ of E having same phase and equal or unequal amplitudes, then the direction of resultant vector 'E' is always linear and hence the polarization is said to be linear polarization.

Let

$$E_x = E_{x0} \cos (\omega t - \beta z)$$
$$E_y = E_{y0} \cos (\omega t - \beta z)$$

Where $E_{x0} > E_{y0}$

The above two equations are shown in the following figure below.



Figure : Representation of E_x and E_y

From above figure, the direction of resultant vector 'E' at different time instants can be observed and shown in the following figure below.



Figure : Linear Polarization

At t = 0, the magnitude of 'E' is greater than at t = T/8 as shown in figure above. At t = T/4, the magnitude is zero and hence no signal shown. Similarly at t = 3T/8, the magnitude is less than at t = T/2. The directions of 'E' at t = 0 and t = T/2 are opposite because of opposite half cycles of signal.

Elliptical polarization:

When the two components $(E_x \text{ and } E_y)$ of E having unequal amplitudes and any phase difference other than zero, then the resultant vector 'E' will trace an elliptical path and hence the polarization is said to be elliptical polarization.

Let

$$E_x = E_{x0} \cos (\omega t - \beta z)$$
$$E_y = E_{y0} \sin (\omega t - \beta z)$$

Where $E_{x0} > E_{y0}$

The above two equations are shown in the following figure below.



Figure : Representation of Ex and Ey

From above figure, the direction of resultant vector 'E' at different time instants can be observed from the following figure below.



63 Figure: Elliptical Polarization

The elliptical polarization can also proved mathematically as follows: Let,

$$E(z) = E_0 e^{-j\beta z} - (1)$$

In above equation only β is included because the wave is assumed to be traveling in free space (lossless medium). To satisfy the definition of elliptical polarization, let

$$E_0 = Aa_x + jBa_y \qquad -(2)$$

Where 'A' and 'B' are the magnitudes of two components of electric field and 'j' represents 90^o phase difference between the two components of electric field intensity. Substitute equation 2 in equation 1

$$E(z) = (Aa_x + jBa_y)e^{-j\beta z}$$

The time varying form of above equation can be written as

$$E(z,t) = Re\left(\left(Aa_{x} + jBa_{y}\right)e^{-j\beta z}e^{j\omega t}\right)$$

As we need to define the polarization at some fixed point, let the fixed point as z = 0, then above equation becomes

$$E(0,t) = Re\left((Aa_x + jBa_y)e^{-j\beta(0)}e^{j\omega t}\right) = Re\left((Aa_x + jBa_y)e^{j\omega t}\right)$$
$$E(0,t) = Re\left((Aa_x + jBa_y)(\cos\omega t + j\sin\omega t)\right)$$
$$E(0,t) = Re(A\cos\omega t a_x + jA\sin\omega t a_x + jB\cos\omega t a_y - B\sin\omega t a_y)$$
$$E(0,t) = A\cos\omega t a_x - B\sin\omega t a_y$$

Therefore,

Equation 3 is equation of ellipse and hence the polarization is said to be elliptical polarization. **Circular polarization:**

When the two components (E_x and E_y) of E having equal amplitudes and 90⁰ phase difference, then the resultant vector 'E' will trace circular path and hence the polarization is said to be circular polarization.

Let

$$E_x = E_{x0} \cos (\omega t - \beta z)$$
$$E_y = E_{y0} \sin (\omega t - \beta z)$$

Where $E_{x0} = E_{y0}$

The above two equations are shown in the following figure below.



From above figure, the direction of resultant vector 'E' at different time instants can be observed from the following figure below.



Figure: Circular Polarization

The circular polarization can also proved mathematically as follows: Let,

$$E(z) = E_0 e^{-j\beta z} \qquad -(1)$$

In above equation only β is included because the wave is assumed to be traveling in free space (lossless medium). To satisfy the definition of circular polarization, let

$$E_0 = E_a a_x + j E_a a_y \qquad - (2)$$

Where E_a represents the magnitude of two components of electric field and 'j' represents 90⁰ phase difference between the two components of electric field intensity. Substitute equation 2 in equation 1

$$E(z) = (E_a a_x + j E_a a_y) e^{-j\beta z}$$

The time varying form of above equation can be written as

$$E(z,t) = Re\left(\left(E_a a_x + jE_a a_y\right)e^{-j\beta z}e^{j\omega t}\right)$$

As we need to define the polarization at some fixed point, let the fixed point as z = 0, then above equation becomes

$$E(0,t) = Re\left(\left(E_a a_x + jE_a a_y\right)e^{-j\beta(0)}e^{j\omega t}\right) = Re\left(\left(E_a a_x + jE_a a_y\right)e^{j\omega t}\right)$$
$$E(0,t) = Re\left(\left(E_a a_x + jE_a a_y\right)(\cos \omega t + j\sin \omega t)\right)$$
$$E(0,t) = Re\left(E_a \cos \omega t a_x + jE_a \sin \omega t a_x + jE_a \cos \omega t a_y - E_a \sin \omega t a_y\right)$$
$$E(0,t) = E_a \cos \omega t a_x - E_a \sin \omega t a_y$$

Therefore,

$$E_x = E_a \cos \omega t$$

$$E_y = -E_a \sin \omega t$$

$$E_x^2 + E_y^2 = E_a^2 \cos^2 \omega t + E_a^2 \sin^2 \omega t = E_a^2 (\cos^2 \omega t + \sin^2 \omega t)$$

$$E_x^2 + E_y^2 = E_a^2 \qquad - (3)$$

Equation 3 is equation of circle and hence the polarization is said to be circular polarization.

In circular polarization, there are two types such as left circular polarization and right circular polarization. When the angle between the two components of 'E' is $+90^{\circ}$, then the resultant vector 'E' will rotate in anti-clock wise direction or left side direction then the polarization is said to be left circular polarization. When the angle between the two components of 'E' is -90° , then the resultant vector 'E' will rotate in clock wise direction or right side direction or right side direction then the polarization is said to be right circular polarization.

UNIT-V (TRANSMISSION LINES)

1. What are the types of transmission lines and explain briefly?

Answer:

Transmission line is cable or channel which will be used to transfer the signal between two points separated by distance. The following are the types of transmission lines

- (i) Coaxial Cable
- (ii) Two-wire line
- (iii) Parallel plate or planar line
- (iv) A wire above the conducting plane
- (v) Microstrip line

Let us discuss the first two types.

Coaxial Cable: The structure of coaxial cable is shown in figure below.



If one wire of open conductor is placed inside a large hallow concentric conductor, the arrangement is called as coaxial cable. Therefore the coaxial cable contains inner conductor and outer conductor as shown in above figure. Equal and opposite currents flows in inner and outer conductors of a coaxial cable. There is a gap between the two conductors and is filled with dielectric medium.

Two-wire transmission line: The structure of two wire transmission line is shown in figure below. Two-wire line contains two parallel wires or conductors spaced with small distance. The gap between the two wires is filled with dielectric medium. The two-wire transmission line is also known as two wire balanced transmission line.



The diameter (d), spacing (S) and length (l) are the dimensional properties of the line are important electrically. For the operation, a load impedance (Z_R) is connected at the receiving end and sinusoidal voltage V_S is applied at the input terminals of the line. Therefore a finite time is needed for the voltage and current to travel the length of the transmission line just as the electromagnetic wave has a finite velocity in space. Truly speaking voltage and current travelling along are accompanied by electromagnetic waves in space between two conductors.

2. What are the primary and secondary parameters of the transmission line and define them?

Answer:

The four line parameters such as Resistance(R), Inductance (L), Conductance (G) and capacitance(C) are referred to as primary parameters or primary constants of the transmission line. These four parameters are defined as follows:

Resistance: It is defined as a loop resistance per unit length. It is sum of the resistances of both the wires per unit length and its units are ohms / km.

Inductance: It is defined as a loop inductance per unit length. It is sum of the inductances of both the wires per unit length and its units are henries / km.

Conductance: It is defined as the shunt conductance between the two conductors per unit length and its units are mhos / km.

Capacitance: It is defined as the shunt capacitance between the two conductors per unit length and its units are farads / km.

Characteristic impedance (Z_0) and propagation constant (γ) are known as secondary parameters or secondary constants of a transmission line. These two parameters are given below:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

3. Derive the basic transmission line equations?

Answer:

The transmission line can be analyzed in terms of voltage and current and the relation between voltage and current can be studied by considering simplest type of transmission line such as parallel wire transmission line as shown in figure below:





Similarly the current change between points 'P' and 'Q' is given by

$$Z_{S} = (R + j\omega L)dx - 1$$
Similarly the total shunt admittance of the short section 'dx' is given by

$$Y_{S} = (G + j\omega C)dx - 2$$
The voltage change between points 'P' and 'Q' is given by

$$V - (V + dV) = I(R + j\omega L)dx$$

$$V - V - dV = I(R + j\omega L)dx$$

$$-\frac{dV}{dx} = I(R + j\omega L) - 3$$
Similarly the current change between points 'P' and 'Q' is given by

$$I - (I + dI) = V(G + j\omega C)dx$$

$$I - I - dI = V(G + j\omega C)dx$$

$$-\frac{dI}{dx} = V(G + j\omega C) - 4$$

Differentiate equation 3 with respect to x,

$$-\frac{d^2V}{dx^2} = \frac{dI}{dx}(R+j\omega L) \qquad -5$$

Substitute equation 4 in equation 5

$$-\frac{d^2V}{dx^2} = -V(G + j\omega C) (R + j\omega L)$$
$$\frac{d^2V}{dx^2} = V(G + j\omega C) (R + j\omega L)$$
$$\frac{d^2V}{dx^2} = \gamma^2 V - 6$$

Where

 $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$ known as propagation constant.

Similarly differentiate equation 4 with respect to x,

$$-\frac{d^2I}{dx^2} = \frac{dV}{dx}(G+j\omega C) \qquad -7$$

Substitute equation 3 in equation 7

$$-\frac{d^{2}I}{dx^{2}} = -I(R + j\omega L)(G + j\omega C)$$
$$\frac{d^{2}I}{dx^{2}} = I(R + j\omega L)(G + j\omega C)$$
$$\frac{d^{2}I}{dx^{2}} = \gamma^{2}I \qquad -8$$

Where $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$ known as propagation constant The equations 6 and 8 are standard linear differential equations and hence their solutions in exponential form is given by

$$V = Ae^{-\gamma x} + Be^{\gamma x} - 9$$

$$I = Ce^{-\gamma x} + De^{\gamma x} - 10$$

4. Derive the equation for the input impedance of the transmission line? Answer:

To derive the input impedance, let us consider the following figure.



We know that,

$$V = V_S \cos h\gamma x - I_S Z_0 \sin h\gamma x - 1$$
$$I = I_S \cos h\gamma x - \frac{V_S}{Z_0} \sin h\gamma x - 2$$

From the figure,

At $x = l, V = V_R$ and $I = I_R$ Substitute equation 3 in equation 1 and 2, then

$$V_R = V_S \cos h\gamma l - I_S Z_0 \sin h\gamma l \qquad -4$$

$$I_R = I_S \cos h\gamma l - \frac{V_S}{Z_0} \sin h\gamma l \qquad -5$$

- 3

- 6

...

IZ

From figure, $V_R = I_R Z_R$ Substitute equations 4 and 5 in equation 6

$$V_S \cos h\gamma l - I_S Z_0 \sin h\gamma l = \left(I_S \cos h\gamma l - \frac{V_S}{Z_0} \sin h\gamma l\right) Z_R$$

Multiply above equation on both sides with Z_0

$$Z_{0}(V_{S}\cos h\gamma l - I_{S}Z_{0}\sin h\gamma l) = Z_{0}Z_{R}\left(I_{S}\cos h\gamma l - \frac{V_{S}}{Z_{0}}\sin h\gamma l\right)$$

$$Z_{0}V_{S}\cos h\gamma l - I_{S}Z_{0}^{2}\sin h\gamma l = Z_{0}Z_{R}I_{S}\cos h\gamma l - V_{S}Z_{R}\sin h\gamma l$$

$$Z_{0}V_{S}\cos h\gamma l + V_{S}Z_{R}\sin h\gamma l = Z_{0}Z_{R}I_{S}\cos h\gamma l + I_{S}Z_{0}^{2}\sin h\gamma l$$

$$V_{S}(Z_{0}\cos h\gamma l + Z_{R}\sin h\gamma l) = I_{S}Z_{0}(Z_{R}\cos h\gamma l + Z_{0}\sin h\gamma l)$$

$$Z_{S} = \frac{V_{S}}{I_{S}} = \frac{Z_{0}(Z_{R}\cos h\gamma l + Z_{R}\sin h\gamma l)}{Z_{0}\cos h\gamma l + Z_{R}\sin h\gamma l} - 7$$

Divide numerator and denominator with $\cos h\gamma l$

$$Z_S = \frac{Z_0(Z_R + Z_0 \tan h\gamma l)}{Z_0 + Z_R \tan h\gamma l} - 8$$

The equations 7 and 8 represent the input impedance of the lossy transmission line. The input impedance for lossless transmission line can be obtained from the above equation by replacing γ with β .

$$Z_{S} = \frac{Z_{0}(Z_{R} + Z_{0} \tan hj\beta l)}{Z_{0} + Z_{R} \tan hj\beta l}$$
$$Z_{S} = \frac{Z_{0}(Z_{R} + jZ_{0} \tan \beta l)}{Z_{0} + jZ_{R} \tan \beta l} - 9$$

5. Derive the characteristic impedance of the transmission line? Answer:

For infinite transmission line the voltage and current are given by

$$V = V_S e^{-\gamma x} - 1$$

$$I = I_S e^{-\gamma x} - 2$$

Differentiate equation1 w.r.t x

$$\frac{dV}{dx} = -\gamma V_S e^{-\gamma x} \qquad -3$$

But

$$\frac{dV}{dx} = -I(R+j\omega L) \qquad -4$$

Substitute equation 3 in equation 4

$$-\gamma V_S e^{-\gamma x} = -I(R + j\omega L)$$

$$\gamma V_S e^{-\gamma x} = I(R + j\omega L) - 5$$

Substitute equation 2 in equation 5

$$\begin{split} \gamma V_S e^{-\gamma x} &= I_S e^{-\gamma x} (R+j\omega L) \\ \gamma V_S &= I_S (R+j\omega L) \end{split}$$

$$Z_0 = \frac{V_S}{I_S} = \frac{(R + j\omega L)}{\gamma}$$

But

$$\begin{split} \gamma &= \sqrt{(R+j\omega L)(G+j\omega C)} \\ Z_0 &= \frac{(R+j\omega L)}{\sqrt{(R+j\omega L)(G+j\omega C)}} \\ Z_0 &= \sqrt{\frac{(R+j\omega L)^2}{(R+j\omega L)(G+j\omega C)}} = \sqrt{\frac{R+j\omega L}{(G+j\omega C)}} \end{split}$$

For lossless transmission line the above equation becomes

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

6. Find out the attenuation constant and phase constant of a transmission line? Answer:

The propagation constant is given by

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{j\omega L \left(1 + \frac{R}{j\omega L}\right) j\omega C \left(1 + \frac{G}{j\omega C}\right)}$$

$$\gamma = \sqrt{j^2 \omega^2 LC \left(1 + \frac{R}{j\omega L}\right) \left(1 + \frac{G}{j\omega C}\right)}$$

$$\gamma = j\omega \sqrt{LC} \sqrt{\left(1 + \frac{R}{j\omega L}\right) \left(1 + \frac{G}{j\omega C}\right)} = j\omega \sqrt{LC} \left(1 + \frac{R}{j\omega L}\right)^{1/2} \left(1 + \frac{G}{j\omega C}\right)^{1/2}$$
where the matrix because

Expand by using binomial series

$$\gamma = j\omega\sqrt{LC} \left[1 + \frac{1}{2}\frac{R}{j\omega L} + \dots \right] \left[1 + \frac{1}{2}\frac{G}{j\omega C} + \dots \right]$$
$$\gamma = j\omega\sqrt{LC} \left[1 + \frac{1}{2}\frac{G}{j\omega C} + \frac{1}{2}\frac{R}{j\omega L} + \dots \right]$$
$$\gamma = j\omega\sqrt{LC} + j\omega\sqrt{LC} \left(\frac{1}{2}\frac{G}{j\omega C} \right) + j\omega\sqrt{LC} \left(\frac{1}{2}\frac{R}{j\omega L} \right)$$
$$\gamma = j\omega\sqrt{LC} + \frac{G}{2}\sqrt{\frac{L}{C}} + \frac{R}{2}\sqrt{\frac{C}{L}}$$
$$\gamma = \frac{G}{2}\sqrt{\frac{L}{C}} + \frac{R}{2}\sqrt{\frac{C}{L}} + j\omega\sqrt{LC} \qquad -1$$

But

$$\gamma = \alpha + j\beta$$
 - 2

Compare equations 1 and 2
$$\alpha = \frac{G}{2} \sqrt{\frac{L}{C}} + \frac{R}{2} \sqrt{\frac{C}{L}}$$
$$\beta = \omega \sqrt{LC}$$

7. Define standing wave ratio and derive the relation between the standing wave ratio and reflection coefficient of a transmission line?

Answer:

When the transmission line terminated with a load impedance (Z_L) which is not equal to characteristic impedance (Z_0) , then there will be reflections from the load. The combination the incident and reflected waves gives new phenomena called standing waves. The standing wave ratio is defined as the ratio of maximum voltage to the minimum voltage or the ratio of maximum current to the minimum current. When standing wave ratio is expressed in terms of voltage then it is known as voltage standing wave ratio (VSWR).

$$VSWR = SWR = S = \frac{V_{max}}{V_{min}}$$
$$V_{max} = V_i + V_r \qquad and \qquad V_{min} = V_i - V_r$$

Where V_i is known as incident voltage and V_r is known as reflected voltage.

The relation between reflection coefficient (Γ) and standing wave ratio (S) is obtained as follows:

$$S = \frac{V_{max}}{V_{min}} = \frac{V_i + V_r}{V_i - V_r}$$

Divide numerator and denominator with V_i

$$S = \frac{1 + \frac{V_r}{V_i}}{1 - \frac{V_r}{V_i}} = \frac{1 + \Gamma}{1 - \Gamma}$$

Where Γ is known as reflection coefficient. From the above equation

$$S(1 - \Gamma) = 1 + \Gamma$$
$$S - S\Gamma = 1 + \Gamma$$
$$S - 1 = \Gamma + S\Gamma = \Gamma(1 + S)$$
$$\Gamma = \frac{S - 1}{S + 1}$$

8. What is a smith chart and explain its uses and applications? Answer:

Smith chart is the most graphical technique used to find the characteristics of a transmission line. Smith chart is also known as polar impedance diagram. It is basically a graphical indication of the impedance of a transmission line and of the corresponding reflection coefficient as one moves along the line. The basic structure of smith chart is shown in figure below.



The smith chart consists of two sets of circles, or arcs of circles. Complete circles whose centers lie on the only straight line on the chart corresponds to various values of normalized resistance and arcs of circles to either side of the straight line corresponds to various values of normalized reactance. The values ($1 \text{ to } \infty$) along the straight line represents the SWR values. The greatest advantage of smith chart is that travel along a lossless line corresponds to movement along a correctly drawn constant SWR circle.

Applications of smith chart:

- (v) Admittance calculation
- (vi) Calculation of SWR along with impedance or admittance.
- (vii) Calculation of length of short circuited piece of transmission line to give required capacitive or inductive reactance.
- (viii) Calculation of length and position of the single stub and double stubs.

9. Write short notes on microstrip lines and give its input impedance?

Answer:

Microstrip lines belong to a group of lines as parallel plate transmission lines. They are widely used in present day electronics such as microwave integrated circuits. Microstrip lines are used as filters, couplers, resonators and antennas. The structure of microstrip line is shown in figure below.



A microstrip line consists of a single ground plane and open strip conductor separated by a dielectric substrate. Analytical derivation of the characteristic impedance of the microstrip line is cumbersome. Compared to conventional transmission lines, microstrip lines have got major advantages such as less weight, lesser volume, smaller size, etc.

The approximate equation for the characteristic impedance of a microstrip line is given by 60 - (8h - w)

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left(\frac{8h}{w} + \frac{w}{4h}\right), & when w/h \le 1\\ \frac{1}{\sqrt{\varepsilon_{eff}}} \frac{120\pi}{\left[\frac{w}{h} + 1.393 + 0.667 \ln\left(\frac{w}{h} + 1.444\right)\right]}, & when w/h \ge 1 \end{cases}$$

Where ε_{eff} is effective dielectric constant or effective relative permittivity, 'h' is the height of the substrate and 'w' is the width of the conducting strip.

$$\varepsilon_{eff} = \frac{(\varepsilon_r + 1)}{2} + \frac{(\varepsilon_r - 1)}{2\sqrt{1 + 12h/w}}$$

10. An air line has a characteristic impedance of 70 Ω and a phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and capacitance per meter of the line

Answer:

Given data:

Characteristic impedance $(Z_0) = 70 \Omega$ Phase constant $(\beta) = 3 \text{ rad/m}$ Frequency (f) = 100 MHzFor lossless line, R = 0 and G = 0

$$Z_0 = \sqrt{\frac{L}{C}}$$
$$\beta = \omega\sqrt{LC}$$
$$\frac{Z_0}{\beta} = \frac{\sqrt{\frac{L}{C}}}{\omega\sqrt{LC}} = \frac{1}{\omega\sqrt{LC}}\sqrt{\frac{L}{C}} = \frac{1}{\omega}\sqrt{\frac{L}{LC^2}} = \frac{1}{\omega C}$$
$$C = \frac{\beta}{\omega Z_0} = \frac{3}{2\pi(100 \times 10^6) \times 70} = 68.2 \ pF/m$$
$$Z_0^2 = \frac{L}{C}$$

$L = Z_0^2 C = (70)^2 \times 68.2 \times 10^{-12} = 334.2 \ nH/m$ 11. A distortionless line has $Z_0 = 60 \ \Omega$, $\alpha = 20 \ mNp/m$, u = 0.6c, where c is the speed of light in a vacuum. Find R, L, G, C and λ at 100 MHz.

Answer:

Given data:

Characteristic impedance $(Z_0) = 60 \Omega$ Attenuation constant (α) = 20 mNp/m Velocity (u) = 0.6c = 0.6 X 3X 10⁸ m/sec Frequency (f) = 100 MHz

Wavelength (
$$\lambda$$
) = $\frac{u}{f} = \frac{0.6 \times 3 \times 10^8}{100 \times 10^6} = 1.8 m$

For distortionless line

$$Z_{0} = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$$

$$\alpha = \sqrt{RG}$$

$$u = \frac{1}{\sqrt{LC}}$$

$$\alpha Z_{0} = \sqrt{RG} \sqrt{\frac{R}{G}} = \sqrt{\frac{R^{2}G}{G}} = R$$

$$R = 20 \times 10^{-3} \times 60 = 1.2 \Omega$$

$$Z_{0}^{2} = \frac{R}{G}$$

$$Z_{0} = \sqrt{\frac{R}{G}}$$

$$G = \frac{R}{Z_{0}^{2}} = \frac{1.2}{(60)^{2}} = 333 \,\mu S/m$$

$$\frac{Z_{0}}{u} = \frac{\sqrt{\frac{L}{C}}}{\frac{1}{\sqrt{LC}}} = \sqrt{\frac{L}{C}} \sqrt{LC} = L$$

$$L = \frac{Z_{0}}{u} = \frac{60}{0.6 \times 3 \times 10^{8}} = 333 \,nH/m$$

$$Z_{0} = \sqrt{\frac{L}{C}}$$

$$C = \frac{L}{Z_{0}^{2}} = \frac{333 \times 10^{-9}}{(60)^{2}} = 92.59 \,pF/m$$

12. A telephone line has R = 30 Ω /km, L = 100 mH/km, G = 0 and C = 20 μ F/km. At f = 1 KHz, obtain

(a) The characteristic impedance of the line

- (b) The propagation constant
- (c) The phase velocity

Answer:

Given data:

$$R = 30 \ \Omega/km, \ L = 100 \ mH/km, \ G = 0, \ C = 20 \mu F/$$

Frequency (f) = 1 KHz

(a) The characteristic impedance is given by

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{30 + j2\pi \times 10^3 \times 100 \times 10^{-3}}{0 + j2\pi \times 10^3 \times 20 \times 10^{-6}}} = 70.75 \angle -1.367^0 \,\Omega$$

(b)

$$\alpha = \frac{G}{2} \sqrt{\frac{L}{C}} + \frac{R}{2} \sqrt{\frac{C}{L}} = \alpha = \frac{0}{2} \sqrt{\frac{100 \times 10^{-3}}{20 \times 10^{-6}}} + \frac{30}{2} \sqrt{\frac{20 \times 10^{-6}}{100 \times 10^{-3}}} = 0.0141 Np/km$$

$$\beta = \omega \sqrt{LC} = 2\pi f \sqrt{LC} = 2\pi \times 10^3 \sqrt{100 \times 10^{-3} \times 20 \times 10^{-6}}$$

$$= 8.885 \ rad/km$$

$$\gamma = \alpha + j\beta = 2.121 \times 10^{-4} + j8.885$$

(c) The phase velocity is given by

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100 \times 10^{-3} \times 20 \times 10^{-6}}} = 707.106 \ km/sec = 7.069 \times 10^5 \ m/sec$$

- 13. A lossless transmission line with $Z_0 = 50 \Omega$ is 30 m long and operates at 2 MHz. The line is terminated with a load $Z_L = 60+j40 \Omega$. If u = 0.6c on the line find
 - (a) The reflection coefficient
 - (b) The standing wave ratio
 - (c) The input impedance

Answer:

Given data:

$$Z_0 = 50\Omega, l = 30 m, f = 2 MHz, Z_L = 60 + j40 \Omega, u = 0.6c$$

(a) The reflection coefficient is given by

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60 + j40 - 50}{60 + j40 + 50} = \frac{10 + j40}{110 + j40} = 0.3523 \angle 56^0$$

(b) The standing ratio is given by

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.3523}{1 - 0.3523} = 2.088$$

(c) The input impedance of lossless line is given by

$$Z_s = \frac{Z_0(Z_L + jZ_0 \tan\beta l)}{Z_0 + jZ_L \tan\beta l}$$

$$\beta l = \frac{\omega l}{\nu} = \frac{\omega l}{0.6c} = \frac{2\pi \times 2 \times 10^6 \times 30}{0.6 \times 3 \times 10^8} = 2.094 \ rad = 2.094 \ \times \frac{180}{\pi} = 120^0$$

$$Z_s = \frac{50((60+j40)+j50\tan(120))}{50+j(60+j40)\tan(120)} = 23.96+j1.35\,\Omega$$

14. A certain microstrip line has a fused quartz ($\varepsilon_r = 3.8$) as substrate. If the ratio of line width to substrate thickness is w/h = 4.5, determine

- (a) The effective relative permittivity
- (b) The characteristic impedance of the line
- (c) The wavelength of the line at 10 GHz

Answer:

Given data:

Relative permittivity (
$$\varepsilon_r$$
) = 3.8
 $\frac{w}{h}$ = 4.5
Frequency (f) = 10 GHz

(a)

$$\varepsilon_{eff} = \frac{(\varepsilon_r + 1)}{2} + \frac{(\varepsilon_r - 1)}{2\sqrt{1 + 12h/w}} = \frac{3.8 + 1}{2} + \frac{3.8 - 1}{2\sqrt{1 + 12/4.5}} = 3.131$$

(b)

$$Z_0 = \frac{1}{\sqrt{\varepsilon_{eff}}} \frac{120\pi}{\left[\frac{W}{h} + 1.393 + 0.667 \ln\left(\frac{W}{h} + 1.444\right)\right]}$$
$$Z_0 = \frac{1}{\sqrt{3.131}} \frac{120\pi}{\left[4.5 + 1.393 + 0.667 \ln\left(4.5 + 1.444\right)\right]} = 30.08 \,\Omega$$
ength is given by

(c) The wavelength is given by

$$\lambda = \frac{v}{f} = \frac{\frac{1}{\sqrt{\mu\varepsilon}}}{\frac{1}{f}} = \frac{\frac{3 \times 10^8}{\sqrt{\mu_r \varepsilon_{eff}}}}{f}$$
$$\lambda = \frac{3 \times 10^8}{f\sqrt{\mu_r \varepsilon_{eff}}} = \frac{3 \times 10^8}{10 \times 10^9 \sqrt{1 \times 3.131}} = 16.9 \ mm$$